Sains Malaysiana 45(1)(2016): 141–156

New Progress in the Study of Intergranular Suction and Shear Strength of Unsaturated Soil

(Kemajuan Baharu dalam Kajian Sedutan Bebutir dan Kekuatan Ricih daripada Tanah Tak Tepu)

LIANSHENG TANG*, HAITAO SANG, LIQUN JIANG & YINLEI SUN

ABSTRACT

The suction between soil particles is the basis and core problem in the study of unsaturated soil. However, is the suction between soil particles just the matrix suction (which has been widely used since the discipline of unsaturated soil mechanics was established). In fact, the concept of matrix suction is from soil science and reflects the water-absorbing capacity of the soil. Matrix suction characterizes the interaction between soil particles and pore water rather than the interactions between soil particles, which were not in conformity with the principle of effective stress of soils. The suction of unsaturated soil, in essence, is the intergranular suction composed of absorbed suction and structural suction. In this paper, first, the basic concepts of absorbed suction and structural suction were briefly introduced. Then, with soil mechanics, powder science, crystal chemistry, granular material mechanics and other related disciplines of knowledge for reference, the quantitative calculation formulas were theoretically deduced for the absorbed suction for equal-sized and unequal-sized unsaturated soil particles with arbitrary packing and the variable structural suction for equal-sized unsaturated soil particles with arbitrary packing and unequal-sized unsaturated soil particles with close tetrahedral packing. The factors that influence these equations were discussed. Then, the shear strength theory of unsaturated soil was established based on the theory of intergranular suction through the analysis of the effective stress principle of unsaturated soil. This study demonstrates that the shear strength of unsaturated soil consists of three parts: The effective cohesive force, the additional strength caused by external loads and the strength caused by intergranular suction. The contribution of the three parts to the shear strength of unsaturated soil depends on the following influence factors: Soil type, confining pressure, water content and density. Therefore, these factors must be comprehensively considered when determining the strength of an unsaturated soil.

Keywords: Absorbed suction; intergranular suction; shear strength; structural suction; unsaturated soil

ABSTRAK

Sedutan antara zarah tanah adalah asas dan teras masalah dalam kajian tanah tidak tepu. Walau bagaimanapun, adakah sedutan di antara zarah tanah hanya sedutan matrik (yang telah digunakan secara meluas sejak disiplin mekanik tanah tak tepu ditubuhkan). Malah, konsep matriks sedutan adalah daripada sains tanah dan mencerminkan keupayaan menyerap air daripada tanah. Matriks sedutan menyifatkan interaksi antara zarah tanah dan air liang daripada interaksi antara zarah tanah yang tidak selaras dengan prinsip tegasan berkesan tanah. Sedutan tanah tak tepu, pada dasarnya, adalah sedutan bebutir terdiri daripada sedutan diserap dan sedutan struktur. Dalam kertas ini, pertama, konsep asas sedutan diserap dan sedutan struktur telah diperkenalkan secara ringkas. Kemudian, dengan mekanik tanah, sains serbuk, kimia kristal, mekanik bahan berbutir dan disiplin lain yang berkaitan pengetahuan untuk rujukan, formula pengiraan secara teori kuantitatif telah disimpulkan untuk sedutan diserap untuk sama bersaiz dan tidak sama rata bersaiz zarah tanah tepu dengan sewenang-wenangnya dan pembungkusan sedutan struktur tanah tak tepu yang berubah-ubah dan yang sama bersaiz zarah dengan pembungkusan sembarangan dan saiz zarah tanah tepu tidak sama rata dengan pembungkusan tertrahedron tertutup. Faktor yang mempengaruhi persamaan ini telah dibincangkan. Kemudian, teori kekuatan ricih tanah tidak tepu ditubuhkan berdasarkan teori sedutan bebutir melalui analisis prinsip tegasan berkesan tanah tidak tepu. Kajian ini menunjukkan bahawa kekuatan ricih tanah tidak tepu mengandungi tiga bahagian: Daya padu efektif, kekuatan tambahan disebabkan oleh beban luaran dan kekuatan yang disebabkan oleh sedutan bebutir. Sumbangan daripada tiga bahagian untuk kekuatan ricih tanah tidak tepu bergantung kepada faktor pengaruh berikut: Jenis tanah, tekanan mengurung, kandungan air dan ketumpatan. Oleh itu, faktor-faktor ini perlu dipertimbangkan secara menyeluruh apabila menentukan kekuatan tanah yang tidak tepu.

Kata kunci: Kekuatan ricih; sedutan bebutir; sedutan struktur; serap sedutan; tanah tak tepu

Introduction

The suction in unsaturated soil is the basis and core problem of the study of the principle of effective stress and shear strength theory of unsaturated soil. Therefore, this subject attracts the attention of scholars from many fields, which leads to the generation of new understanding. The matrix suction, which is widely used at present, reflects the water-absorbing capacity of the soil and characterizes the interaction between soil particles and pore water rather than the interactions between soil particles, which is not in conformity with the principle of effective stress of soils (Tang et al. 2006). Shen (1996a, 1996b) insists that although suction increases the resistance against sliding between soil particles, the soil's ability to resist deformation does not proportionally increase with the increase in suction. Therefore, the concept of reduced suction (effective suction, or the portion of suction that effectively increases the soil's strength and resistance to deformation) is put forward. Then, a generalized suction model is put forward to include all the forces that can increase the sliding resistance force between particles, such as matrix suction, bonding force, occlusion force and viscous force. Tang (2000a) successively put forward the concepts of absorbed suction and structural suction and concluded that the suction of unsaturated soil, in essence, is the intergranular suction composed of absorbed suction and structural suction and that using the matrix suction (i.e. the soil's capacity to absorb water) to represent intergranular suction, which includes variable structural suction (for example, the bonding force between particles which changes with the water content) is a fundamental mistake in classic unsaturated soil mechanics.

Willett et al. (2000) tested the capillary force between two equal or unequal sizes of spherical particles but did not provide the theoretical perspective. Shimada et al. (2000) deduced the additional effective stress of equal-sized particles caused by matrix suction of soil and interfacial tensions, but they mistakenly believe that the matrix suction parallel to the direction of its attachment and the total intergranular additional effective stress are generated by the matrix suction and interfacial tensions, not realizing that the matrix suction is a component of interfacial tensions. Miao et al. (2001) and Mu et al. (2004) put forward the concept of additional internal stress and argue that the matrix suction can be decomposed into capillary suction and additional internal stress. Although Miao et al. (2001) and Mu et al. (2004) observed that the component force of interfacial tensions in the direction of the center of two sphere can increase the strength of soil, the matrix suction and the additional internal stress must still be added as components of the effective stress when calculating the effective stress, or else the component and original forces will be mistakenly summed up. Jia et al. (2005) deduced the intergranular additional effective stress produced by the interfacial tensions of arbitrarily arranged unequal-sized particles of unsaturated soil, but they perpetuated the error of Shimada et al. (2000). The suction stress discussed by Lu and Likos (2006) refers to the net intergranular force acting on unsaturated soil particles comprehensively caused by the negative pore water pressure and interfacial tensions, which is consistent with the literature (Tang & Wang 2006). However, the important role of variable structural suction in unsaturated soil has not received enough attention, which shows that Luan et al. (2006) have attached great importance to the tensile stress effect of the component force of interfacial tensions in the direction of the line joining the centers of two soil particles and it is a pity that they still have not completely escaped from the basic idea of matrix suction. Zhao et al. (2007) suggested that the total suction of unsaturated soil is just the disjoining pressure of aqueous films, based on the aqueous film theory, which includes electrostatic forces, the van der Waals force, capillary tension, matrix suction and osmotic suction. However, these authors mistakenly add them up as the total suction due to not fully understanding the relationships between these suctions. The relationship between suction and the volume of water between soil particles with different radii was developed by He et al. (2010). This relationship provides microscopic hysteresis mechanisms in soil-water characteristic curves, but a discussion of the nature of the matrix suction is not provided. Gens (2010) indicates that the matrix suction is approximately composed of a capillary suction part and an adsorption part and the part that dominates is dependent on the water content and soil type. However, the adsorption part just belongs to the component of structural suction, as developed by Tang (2000a), whose mechanism is quite different from matrix suction. Zhang et al. (2013, 2007) proposed the theoretical model for the interaction between wet particles and liquid bridges, recognizing unsaturated soil as wet particulate material, but they mistakenly believe that the capillary force is caused by matrix suction and interfacial tensions.

Tang and his leading research team explore the quantitative calculation of absorbed suction and structural suction and have successively presented the quantitative calculation methods for absorbed suction in cubic, loose packing and structural suction in typical packing (Tang et al. 2013). The latter experiments incorporated equal-sized soil particles in several typical packing styles. However, the quantitative calculation method for equal-sized soil particles in arbitrary packing and unequal-sized soil particles in arbitrary packing have not been developed. On the basis of the basic conception of absorbed suction and structural suction, along with soil mechanics, powder science, crystal chemistry, granular material mechanics and other related disciplines of knowledge for reference, we theoretically deduced the quantitative calculation formulas of the absorbed suction for equal-sized and unequal-sized unsaturated soil particles in arbitrary packing; and the variable structural suction for equal-sized unsaturated soil particles in arbitrary packing and unequal-sized unsaturated soil particles in tetrahedral close packing. The influence factors on these equations are discussed and a shear strength theory of unsaturated soil is established based on the theory of intergranular suction through the analysis of the effective stress principle of unsaturated soil.

MATERIALS AND METHODS

BASIC CONCEPTS OF INTERGRANULAR SUCTION

The research on the principle of effective stress and the strength theory of unsaturated soil must be based on the correct understanding of suction in unsaturated soil. However, for a long time, there has been a misunderstanding of the relationship between intergranular suction and matrix suction of unsaturated soil. Often, matrix suction is assumed to be the sum of various suctions between soil particles without analysis of the various suctions in the unsaturated soils and their characteristics. The intergranular suction, proposed by Tang (2006) refers to the suctions that act on soil particles and contribute to the interaction between soil particles, including absorbed suction and structural suction. Structural suction can be divided into intrinsic structural suction and variable structural suction according to whether it is affected by water content change, as shown in Figure 1.

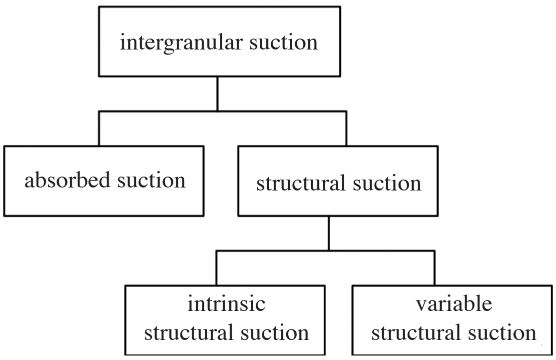


FIGURE 1. Intergranular suction composition diagram

Interfacial tensions, a typical phenomenon of gas-liquid interfaces, appears in unsaturated soil pores and affects the shear strength of soil through a contractile skin along the gas-liquid interface. The horizontal tangential component of interfacial tensions makes the soil particles produce an additional pressure stress, as shown in Figure 2, which directly act on the soil particles and influences the strength and deformation of the soil. This stress represents a part of the effective stress of unsaturated soil, referred to as absorbed suction in this paper. The vertical component of interfacial tensions is the classical matrix suction, which does not act on the soil particles and does not increase the additional effective stress between the soil particles. This suction does not contribute to the strength of the soil mass. However, the matrix suction is popularly used as part of the effective stress of unsaturated soils, which essentially violates the principle of the effective stress of soils.

The existing research showed that the macroscopic mechanical properties of soil were closely related with its structure. The basic structure model of unsaturated soil includes two parts: the soil skeleton and the fluid within it. The soil skeleton is a porous structure with different connection strengths formed by soil particles and their different packing types and the fluid (gas and liquid) fills the skeleton pores in different forms. Therefore, an unsaturated soil always has a certain structural strength, and this strength is equivalent to the strength due to structural suction. When the internal stress caused by an external load acting on the unsaturated soil is less than the strength of the soil skeleton, the external load will be mainly borne by the soil skeleton, which can prevent the deformation and failure of the soil. Even if the internal stress caused by the external load is larger than the strength of soil skeleton, the soil skeleton will, together with its internal pore fluid, prevent the deformation and failure of soil. Therefore, the structural strength or structural suction is part of the effective stress of the unsaturated soil.

BASIC CONCEPTS OF ABSORBED SUCTION

According to different research angles, absorbed suction can be divided into microscopic absorbed suction (p_s) and macroscopic absorbed suction (s_a). The former, p_s, describes the microscopic state of the absorbed suction existing between soil particles due to the interfacial tensions caused by the contractile skin at the air-water interface. The latter, s_a, describes the macroscopic state of the absorbed suction on the surface of a range of soil particles, which averages the microscopic absorbed suction across the same range of soil particles. Obviously, the macroscopic absorbed suction, s_a, is more favorable in studying the effective stress and the shear strength of unsaturated soil, but the microscopic absorbed suction, p_s, must be developed first.

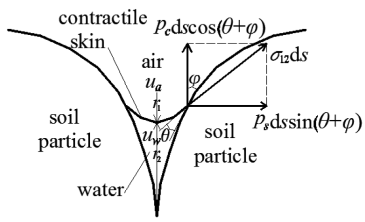


FIGURE 2. Schematic diagram of the mechanical decomposition of the interfacial tensions at the air-water interface

One typical element of the air-water interface in the three-phase system of air-water-soil in unsaturated soil is shown in Figure 2, where σ_{12} is the interfacial tensions coefficient (interfacial tensions of per unit length). The Young-Laplace equation provides a general relationship between matrix suction and the interface geometry, which may be written as,

$$u_a - u_w = \sigma_{12}\left(\frac{1}{r_1} - \frac{1}{r_2}\right), \tag{1}$$

where u_a and u_w are the air and water phase pressures, respectively, the difference $(u_a - u_w)$ is the matrix suction and r_1 and r_2 are the two radii of the curvature of the interface.

As shown in Figure 2, the vertical force $\sigma_{12}ds \cos(\theta + \varphi)$ due to interfacial tensions $\sigma_{12}ds$ on the small segments ds balances the matrix suction $(u_a - u_w)$ in the vertical direction.

$$u_a - u_w = \sigma_{12}ds \cos(\theta + \varphi). \tag{2}$$

From (2), the matrix suction $(u_a - u_w)$ is the external performance (output) of soils in the vertical direction caused by interfacial tensions under external loads (inputs). The result is just a response to the external load and it does not act on the soil particles and makes no contribution to the strength of soils. This demonstrates that it is unreasonable to view the matrix suction as part of the effective stress of unsaturated soil, which cannot reflect the mechanics essence of the effective stress principle. However, the horizontal force $\sigma_{12}ds \cos(\theta + \varphi)$ due to interfacial tensions $\sigma_{12}ds$ on the small segments ds makes the soil particles produce an additional pressure stress, which acts directly on the soil particles. Under equilibrium conditions, the water in the meniscus between the two spherical particles produces a tensile stress on the soil particles, pulling the adjacent soil particles towards each other. The tensile stress acts directly on the soil particles and has a direct impact on the strength and deformation of the soil. This effect is an important part of the effective stress and is referred to as the microscopic absorbed suction, p_s. Balancing the two above forces leads to,

$$p_s = \sigma_{12}ds \cos(\theta + \varphi). \tag{3}$$

Thus far, the definition of absorbed suction can be clearly understood as the suction produced by interfacial tensions between the soil particles due to wetting using water. The formation of absorbed suction is a result of the interaction of three phases (solid, liquid, and gas) in the soil. The absorbed suction characterized by tensile stress is equal to the horizontal component of interfacial tensions. For example, assuming that two spherical particles have a certain gap and that water occupies this gap, capillary water will come into being in contact of the two spherical particles and the two spherical particles will be drawn together.

BASIC CONCEPTİON OF STRUCTURAL SUCTİON

Baker and Frydman (2009) thought that it was impossible that water tension (the capillary potential under field conditions, when $u_a = 0$) is greater than a certain threshold value (in the range of 100 to 400 kPa) due to cavitation by testing and that it is impossible for the matrix suction to be greater than this threshold value in practical field conditions due to the cavitation of negative pore water. Hence, what does it mean when the matrix suction is greater than this threshold value? The current constitutive models of unsaturated soils are mostly based on the axis translation technique, which eliminates the possibility of cavitation. Therefore, what is the relevance of these models to field conditions with atmospheric air pressure? Some other suctions, definitely other than matrix suction, must be considered to solve the above problems. Zhao et al. (2013) argued that it is adsorption, consistent with Baker and Frydman (2009) that was produced by the physical-chemical interactions among all the phases (water, air and soil), including the long-range electrostatic force (electric double layer theory) and the short-range van der Waals force and other hydration forces. In fact, adsorption is just a component of structural suction. Mitchell and Soga (2005) defined the soil structure as the combined effects of the fabric, mineral components and interactions among particles. Additionally, Zhao et al. (2013) thought that soil structure should include the distributions of pore water and pore air and the interactions among all the phases (e.g. capillary and physical-chemical forces). The test of Wang (1987) and Zhang et al. (2006) proved that the clay mineral composition, bound water and their interaction with the aqueous solution are the basis of determining the structural characteristics of cohesive soil. Therefore, this paper holds that the structure of soil should also include its compositional characteristics, such as the mineral composition of soil particles, the chemical composition of pore fluid, the ice in frozen soil and so on. Thus, Tang, as early as 2000b, called for geotechnical circles to pay more attention to the impact of water-soil chemical action on the physical and mechanical properties and the structure of soil, especially in the study of unsaturated soils (Tang 2000b). The geotechnical chemical mechanical discipline was first proposed in China by this author.

Based on the above existing study, the structural suction proposed by Tang refers to the sum of all tensile stresses related to the properties of soil structure, including the cementing force, electrostatic force, magnetic force and occluding force. All the above forces are affected by water content, the soil particle packing method and the chemical composition of the pore solution, but the degree of influence differs (Tang 2000b). The structural suction reflects the structural characteristics of the soil, including the soil's skeleton characteristics, porosity characteristics, intergranular characteristics and component characteristics. As mentioned above, structural suction can be divided into intrinsic structural suction and variable structural suction according to whether it is affected by changes in the water content. Their difference in mechanical properties is obvious. The intrinsic structural suction is the intrinsic cohesion between soil particles, which does not change with water content and corresponds to a certain original structure of the soil. This suction is the structural suction that still exists between soil particles when the soil is fully saturated (hence, the intrinsic structural suction is also called the saturated structural suction) and it cannot be recovered once lost. The variable structural suction varies with water content, grain packing modes and cement chemical properties. This suction's properties are recoverable if lost, which means the variable structural suction can recover gradually in response to changes in the external conditions. For example, the cementing force is an important part of effective stress of unsaturated soil and is seriously affected by water content and cement chemistry. The tests of Baker and Frydman (2009) showed that adsorption still accounts for a large proportion in the matrix suction when the clay has a high degree of saturation, which proves that the structural suction has a great impact on the suction and strength of cohesive soil and indirectly proves the validity of structural suction. Meng (2013) assumes that the contact between the soil particles (the carrier of the effective stress in unsaturated soil) and the combined water film (combined phase) is the fifth phase and then introduced a general formula of atmospheric tension for the effective stress principle of unsaturated soil. This model successfully explained the laboratory tensile strength tests of an expansive soil in the literature (Ran et al. 2011). In fact, the fifth phase is just a component of the intrinsic structural suction. The suction in unsaturated soil includes not only absorbed suction and matrix suction due to interfacial tensions on the air-water interface but also structural suction, including the suction due to combined water.

Results and Discussion
Quantitative Calculation of Intergranular Suction

Quantitative calculation of absorbed suction

Absorbed suction of equal-sized soil particles with arbitrary packing The subject of single liquid bridges between two neighbouring sphere of equal size has drawn much attention (Kim & Hwang 2003; Lian et al. 1993; Willet et al. 2003), however, the Laplace equation was used as the governing relation between the matric suction and the mean curvature of the meniscus, ignoring the fact that the water-induced attractive interaction is microscopic absorbed suction rather than matric suction.

In order to simplify the derivation process, the study object is assumed to be as follows before the formula is derived: soil particles are ideally spherical; pore-water is evenly distributed between the soil particles; the pore-water between particles is a saddle warp in the three dimensional space; and the influence of gravity on the contractile skin is neglected.

The projection geometry of the air-water-solid interface is shown in Figure 3, where R is the radius of the soil particle, θ is the contact angle between the contractile skin and the particles, r_1 is the lateral curvature radius of the meniscus gaseous phase, r_2 is the curvature radius of the liquid phase, φ is the particle saturated angle, T_s is the interfacial tensions coefficient and $2d$ is the distance of the particles.

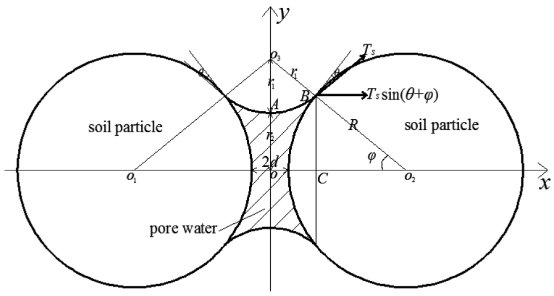


FIGURE 3. The geometric relationships of particles with equal diameter

The interfacial tensions are decomposed into two components, the horizontal direction and the vertical direction. The horizontal balance relationship is as follows:

$$T_s \sin(\varphi + \theta)\cdot 2\pi R \sin\varphi = p_s.\pi (R \sin \varphi)^2. \tag{4}$$

The microscopic absorbed suction p_s can then be computed as follows:

$$p_s = \frac{2T_s \sin(\varphi+\theta)}{R\sin\varphi}. \tag{5}$$

From (5), when soil is in a certain state, the radius of soil particle (R), the contact angle between the contractile skin and the particles (θ) and the interfacial tensions coefficient (T_s) are certain; hence, the microscopic absorbed suction, p_s, depends only on the particle saturated angle, φ.

According to the symmetrical geometric relationship in Figure 3, assuming the volume of pore water is V_0, the volume of trapezoid *OABC* revolving around the axis O_1O_2 is V_1, the volume of spherical cap is V_2, the relationship of the volumes can be written as follows,

$$V_0 = 2(V_1 - V_2), \tag{6}$$

where V_1 is determined as follows:

$$\begin{aligned} V_1 &= \int_0^s \pi\left(r_1 + r_2 - \sqrt{r_1^2 - x^2}\right)^2 dx \\ &= \pi[s(r_1+r_2)^2 - \frac{1}{3}s^3 - r_1^2(r_1+r_2), \\ &\times(\arcsin\frac{s}{r_1} + \frac{s\sqrt{r_1^2 - s^2}}{r_1^2})] \end{aligned} \tag{7}$$

and V_2 is determined as follows:

$$\begin{aligned} V_2 &= \int_0^s \pi(\sqrt{R^2 - (x-R-d)^2})^2 \\ &= \frac{\pi R^3}{3}(2+\cos\varphi)(1-\cos\varphi)^2, \end{aligned} \tag{8}$$

where s is the abscissa of the highest point of the upper part of the concave liquid surface, $s = d + R(1- \cos\varphi)$; r_1 is the lateral curvature radius of the meniscus gaseous phase, $r_1 = \frac{R+d}{\cos\varphi} - R$; and r_2 is the curvature radius of the liquid phase, $r_2 = R - \frac{(R+d)(1-\sin\varphi)}{\cos\varphi}$.

Substituting (7) and (8) into (6) yields the relationship of the water ring volume and saturated angle as follows:

$$\begin{aligned} V_0 &= 2(\frac{1}{3}\pi R^3(2-3\cos\varphi+\cos^3\varphi) - \frac{1}{3}\pi(d+R-\cos\varphi)^3 \\ &+\pi(d+R-R\cos\varphi)(d+R)^2\tan^2\varphi + \pi(d+R-R\cos\varphi) \\ &\times[R-(d+R)\sec\varphi]^2 - \pi(d+R)[R-(d+R)\sec\varphi]^2 \\ &\times\tan\varphi(\frac{\pi}{2}-\varphi+\sin\varphi\cos\varphi)\}. \end{aligned} \tag{9}$$

When the distance of particles $2d$, is zero, (7) can be written as follows:

$$V_0 = 2\pi R^3(1-\sec\varphi)^2[1-(\frac{\pi}{2}-\varphi)\tan\varphi]. \tag{10}$$

For convenience, (9) and (10) also can be abbreviated as follows:

$$V_0 = f_1(\varphi), \tag{11}$$

where f_1 is the function of φ.

Assuming the soil particles number in unit volume is N_s, the total volume of the soil pore water can be written as follows:

$$V_w = V_0 \cdot N_c \cdot \frac{1}{2} \cdot N_s. \tag{12}$$

According to the relationships among the three-phase physical indicators of the soil, the volume of pore water in the soil can be written as follows:

$$V_w = \frac{m_s w}{\rho_w} = D \cdot w \cdot \frac{4}{3}\pi R^3 \cdot N_s, \tag{13}$$

where D is the relative density, $D = \frac{\rho_s}{\rho_w}$.

Combining (12) and (13) leads to the volume of pore water, as follows:

$$V_0 = \frac{8}{3}\pi R^3 \cdot D \cdot w \cdot \frac{1}{N_c}, \tag{14}$$

where the relative density, D, is constant for a specific soil. Therefore, the volume of pore water depends only on the water content and the coordination number.

By the Smith equation of powder mechanics (Ibrahim et al. 2014), the relationship of the coordination number and void ratio can be written as follows:

$$\frac{e}{1+e} = \frac{0.414N_c - 6.527}{0.414N_c - 10.968}. \tag{15}$$

According to the relationships between the three-phase physical indicators of soil, the relationship between the void ratio and dry density can be written as follows:

$$e = \frac{V-V_s}{V_s} = \frac{\rho_s}{\rho_d} - 1, \tag{16}$$

where ρ is particle density which varies slightly for different types of soil and can be obtained from tables in relevant codes and the parameter ρ_d is the dry density of the soil.

Combining (14), (15) and (16) yields the relationship between the volume of pore water and the water content as follows:

$$V_0 = \frac{\frac{8}{3}\pi R^3 \cdot D \cdot w}{26.493 - 10.927\frac{\rho_s}{\rho_d}}. \tag{17}$$

For convenience, (17) can also be abbreviated as follows:

$$V_0 = f_2(w, \rho_s, \rho_d), \tag{18}$$

where f_2 is a function of w, ρ_s and ρ_d.

Combining (11) and (18) leads to the relationship between the saturated angle, water content and dry density:

$$\varphi = g(w, \rho_d), \tag{19}$$

where g is a function of w and ρ_d.

With substitutions into (5), the microscope absorbed suction, in kPa, of equal-sized unsaturated soil particles with arbitrary packing is obtained as follows:

$$p_s = \frac{2T_s\sin[g(w,\rho_d)+\theta]}{R\sin[g(w,\rho_d)]}. \tag{20}$$

Through the above derivation, the expression of microscopic absorbed suction, in kPa, with the macroscopic state parameters, including the particle radius R, water content w and dry density ρ_d, was established. Because (20) is complicated, the microscopic absorbed suction must be solved via computer programming. Then, the microscopic absorbed suction, F_s, in N (Newton), can be obtained as follows via the multiplication of p_s by the action area:

$$\begin{aligned} F_s &= p_s \cdot \pi(R\sin\varphi)^2 \\ &= \frac{2T_s\sin[g(w,\rho_d)+\theta]}{R\sin[g(w,\rho_d)]} \cdot \pi[R\sin g(w,\rho_d)]^2. \end{aligned} \tag{21}$$

The expression of microscopic absorbed suction, in N, with the macroscopic state parameters, including the particle radius R, water content w and dry density ρ_d, is thus established.

For the soil in a unit volume, the number of total contact points of soil particles is defined as N_v and the coordination number N_c refers to the number of contact points of a single particle with all its surrounding particles. Hence, N_v is the product of the soil particles, N_s, in a unit volume of soil and the coordination number, N_c. Because each contact point is common to two particles, the total contact number in unit soil volume can be written as follows:

$$N_v = \frac{1}{2} N_s \cdot N_c. \tag{22}$$

The total surface area of soil particles in a unit volume is as follows:

$$S = N_s \cdot S_s = N_s \cdot 2\pi R^2, \tag{23}$$

where S_s is the surface area of a single particle; and R is the radius of a single particle.

The contact points in a unit volume can be written as follows:

$$N = \frac{N_v}{S} = \frac{N_c}{4\pi R^2}. \tag{24}$$

The relationship between macroscopic absorbed suction, s_a, in kPa and microscopic absorbed suction, F_s, in N (Newton), can be written as follows:

$$s_a = Fs \cdot N. \tag{25}$$

Substituting (21) and (24) into (25) yields the macroscopic absorbed suction, s_a, in kPa, of equal-sized soil particles:

$$s_a = \frac{2T_s \sin[g(w,\rho_d)+\theta]}{R\sin[g(w,\rho_d)]} \cdot \pi\left[R\sin g(w,\rho_d)\right]^2 \cdot \frac{N_c}{4\pi R^2}. \tag{26}$$

This expression establishes the macroscopic absorbed suction, s_a (in kPa), of equal-sized soil particles with macroscopic state parameters, including the particle radius (R), water content (w), dry density (ρ_d) and coordination number (N_c).

Absorbed suction of unequal-sized soil particles with arbitrary packing It is worth noting that, almost all the particulate systems encountered in actual practice consist of rough particles with different sizes. Therefore, the system of two even-sized particles becomes the basis and much pioneering work give attention to it (Batool et al. 2015). However, these studies provide insightful information regarding liquid retention and interparticle force in unsaturated granular materials using the Laplace equation, do not consider microscopic absorbed suction.

The assumption in this condition is the same as that of the equal-sized soil particles. The geometric relationships of the projected planes of the particles are shown in Figure 4, where R is the radius of the large soil particle; r is the radius of the small particle; θ is contact angle between contractile film and the particles; r_1 is the lateral curvature radius of the meniscus gaseous phase; r_2 is the curvature radius of the liquid phase; α is the saturated angle of the large particle; β is the saturated angle of the small particle; T_s is interfacial tensions coefficient; and $2d$ is the distance between the particles.

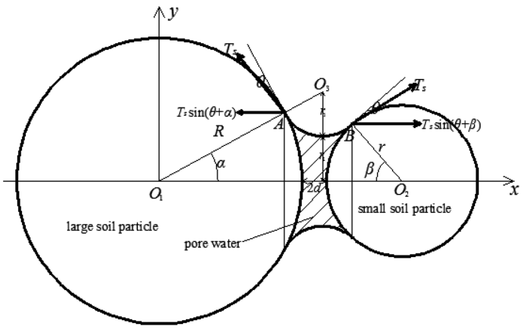


FIGURE 4. The geometric relationships of particles with unequal diameters

The horizontal balance of the large soil particle and the small soil particle is written as follows:

$$T_s \sin(\alpha + \theta).\ 2\pi R \sin \alpha = p_{s1} \cdot \pi\,(R \sin \alpha)^2, \tag{27}$$

$$T_s \sin(\beta + \theta).\ 2\pi r \sin \beta = p_{s2} \cdot \pi\,(r \sin \beta)^2, \tag{28}$$

where p_{s1} and p_{s2} are the microscopic absorbed suction forces acting on the large and small soil particle, respectively.

Equations (27) and (28) can be rewritten as follows:

$$p_{s1} = \frac{2T_s \sin(\alpha+\theta)}{R\sin\alpha}, \tag{29}$$

$$p_{s2} = \frac{2T_s \sin(\beta+\theta)}{R\sin\beta}. \tag{30}$$

From (30), when soil is in a certain state, the radius of soil particle R, the contact angle between the contractile skin and the particles θ and the interfacial tensions coefficient T_s are constant; hence, the microscopic absorbed suction, p_s, depends only on the particle saturated angles α and β.

The overall horizontal balance, including the large soil particle, the small soil particle and the pore water, can be written as follows:

$$T_s \sin(\alpha + \beta).\ 2\pi R \sin \alpha = T_s \sin(\beta + \theta).\ 2\pi r \sin \beta. \tag{31}$$

Equation (31) can be rewritten as follows:

$$\frac{R}{r} = \frac{\sin(\beta+\theta)\sin\beta}{\sin(\alpha+\theta)\sin\alpha}. \tag{32}$$

Assuming that the volumes of the spherical crowns of the large and small soil particles wetted with water are V_1

and V_2, respectively and that the volume of the trapezoid ABCD revolving round the x axis is V, the volume of the pore water, V_0, can be obtained with following formula:

$$V_0 = V - V_1 - V_2, \tag{33}$$

where V can be calculated as follows:

$$\begin{aligned} V &= \int_{R\cos\alpha}^{R+d+r-r\cos\beta} \pi\left(n - \sqrt{r_1^2 - (x-m)^2}\right)^2 dx \\ &= \pi(b-a)[m(b+a) + (n^2 + r_1^2 - m^2) - \frac{b^2+ab+b^2}{3} \\ &\quad -\pi n[(b-m)\sqrt{r_1^2-(b-m)^2} - (a-m)\sqrt{r_1^2-(a-m)^2}\,] \\ &\quad -\pi n r_1^2\left(\frac{\arcsin(b-m)}{r_1} - \frac{\arcsin(a-m)}{r_1}\right), \end{aligned} \tag{34}$$

where m and n are the abscissa and ordinate, respectively, of the center of circle O_3,

$$m = R\cos\alpha + \frac{R+r+d+(R-r)\cos\beta}{\cos\alpha+\cos\beta}\cos\alpha,$$

$$n = R\sin\alpha + \frac{R+r+d+(R-r)\cos\beta}{\cos\alpha+\cos\beta}\sin\alpha;$$

r_1 is the radius of circle O_3,

$$r_1 = \frac{R+r+d+(R-r)\cos\beta}{\cos\alpha+\cos\beta};$$

and a and b are the projections of points A and B^b, respectively, in the x axis, $a = R\cos\alpha$, $b = R + d + r(1 - \cos\beta)$。.

With (6) as reference, V_1 and V_2 can be obtained as follows:

$$V_1 = \frac{\pi R^3}{3}(2+\cos\alpha)(1-\cos\alpha)^2, \tag{35}$$

$$V_2 = \frac{\pi r^3}{3}(2+\cos\beta)(1-\cos\beta)^2. \tag{36}$$

Substituting (34), (35) and (36) into (33) yields:

$$\begin{aligned} V_0 &= \pi(b-a)[m(b+a) + (n^2 + r_1^2 - m^2) - \frac{b^2+ab+b^2}{3}] \\ &\quad -\pi n[(b-m)\sqrt{r_1^2-(b-m)^2} - (a-m)\sqrt{r_1^2-(a-m)^2}\,] \\ &\quad -\pi n r_1^2\left[\frac{\arcsin(b-m)}{r_1} - \frac{\arcsin(a-m)}{r_1}\right] - \\ &\quad \frac{\pi R^3}{3}(2+\cos\alpha)(1-\cos\alpha)^2 - \frac{\pi R^3}{3}(2+\cos\beta)(1-\cos\beta)^2. \end{aligned} \tag{37}$$

For convenience, (37) can also be abbreviated as follows:

$$V_0 = f_3(\alpha, \beta), \tag{38}$$

where f_3 is a function of α and β.

The volume of pore water between the soil particles from microscopic factors is equal to the water content from macro factors. Therefore,

$$V_0 \times N_c \times \frac{1}{2}N_s = D \times w \times \left(\frac{4}{3}\pi R^2 \times N_{s1} + \frac{4}{3}\pi r^3 \times N_{s2}\right), \tag{39}$$

where N_s, N_{s1} and N_{s2} are the number of total soil particles, large soil particles and small soil particles, respectively ($N_s = N_{s1} + N_{s2}$).

Assuming $N_{s1} = \lambda N_s$, (39) can be rewritten as follows:

$$\frac{1}{2}V_0 N_c = \frac{4}{3}\pi D w[\lambda R^3 + (1-\lambda)r^3]. \tag{40}$$

If $R = r$, particles with unequal diameters turn into particles with equal diameters, and (40) can be abbreviated as (14), which proves the validity of (40). This correlation indicates (40) is the general formula for calculating the volume of pore water, which is applicable to particles with unequal diameters and particles with equal diameters.

By the Rumf formula in powder mechanics (Batool et al. 2015; Ibrahim et al. 2014), the coordination number N_c can be written as follows:

$$N_c = \frac{1+e}{e}\pi = \frac{\rho_s}{\rho_s + \rho_d}\pi. \tag{41}$$

Therefore, the volume of pore water can be calculated as follows:

$$V_0 = \frac{2}{3}Dw[\lambda R^3 + (1-\lambda)r^3](1 - \frac{\rho_d}{\rho_s}). \tag{42}$$

For convenience, (42) can be abbreviated as follows:

$$V_0 = f_4(\lambda, w, \rho_d), \tag{43}$$

where f_4 is a function of λ, w and ρ_d.

Combining (32), (38) and (43) can yield the relationships between the saturated angle (α, β), the ratio of the number of large and small soil particles (λ), water content (w) and dry density (ρ_d):

$$(\alpha, \beta) = h(\lambda, w, \rho_d), \tag{44}$$

where h is a function of λ, w and ρ_d.

Via substitution of the saturated angle α and β into (29) or (30), the microscopic absorbed suction, in N, of equal-sized unsaturated soil particles is obtained as follows:

$$\begin{aligned} F_s &= T_s \sin(\alpha + \theta) \times 2\pi R \sin\alpha \\ &= T_s \sin(\beta + \theta) \times 2\pi r \sin\beta. \end{aligned} \tag{45}$$

Assuming N_s, N_{s1} and N_{s2} are the number of all soil particles, large soil particles and small soil particles, respectively, in a unit volume where $N_s = N_{s1} + N_{s2}$; $N_{s1} = \lambda N_s$; and S_{s1} and S_{s2} are the superficial areas of large soil particles and small soil particles, respectively ($S_{s1} = 2\pi R^2$; $S_{s2} = 2\pi r^2$), the number of contact points of soil particles in a unit area is as follows:

$$N = \frac{N_v}{S} = \frac{\frac{1}{2}N_c \times N_s}{N_{s1} \times S_{s1} + N_{s2} \times S_{s2}}. \tag{46}$$

The relationship between macroscopic absorbed suction, S_a, in kPa and microscopic absorbed suction, F_s, in N, can be written as follows:

$$S_a - F_s \times N. \tag{47}$$

Substituting (46) into (47) yields the macroscopic absorbed suction, s_a, in kPa, of unequal-sized soil particles as follows:

$$s_a = F_s \times \frac{\rho_s}{4(\rho_s - \rho_d)(\lambda R^2 + (1-\lambda) r^2]}. \tag{48}$$

This expression establishes the macroscopic absorbed suction, s_a, in kPa, of unequal-sized soil particles with the macroscopic state parameters, including the ratio of the number of large and small soil particles (λ), the large particle radius (R), water content (w), dry density (ρ_d) and coordination number N_c.

A difference between the radius of particles results in a difference in the saturated angle formed by the soil grains wetted with water, which causes changes in the microscopic absorbed suction. Compared with the packing of equal-sized soil particles, the nonlinear nature of the formula for absorbed suction is greater and more complicated. Meanwhile, the size of the absorbed suction is related to not only the particles radius but also the ratio of the number of large and small particles. With increasing numbers of variables, the solution becomes more difficult compared with the packing of equal-sized soil particles.

Discussion on the influence factor of absorbed suction

According to the above derivation process, the influence factors of macroscopic absorbed suction (absorbed suction for short) include the coefficient of interfacial tensions, saturated angle, contact angle, particle radius, dry density, soil density, water density, water content and degree of saturation. The interaction rules between absorbed suction and the above influence factors should not be different when the particle sizes are equal or unequal (Ashraf et al. 2013). The distinction is the different degree of absorbed suction in the numerical values in these two cases, in which the dry density and water content changes. However, the slopes of their curves, i.e. rising or falling, are different. For the packing of the two sets of unequal-sized particles, there shall always be a corresponding packing of equal-sized particles, which can be written as $R_m^2 = \lambda R^2 + (1 - \lambda) r^2$. Thus, the packing of the two sets of unequal-sized particles can be unified in numerical value as the packing of equal-sized particles. However, this equivalence cannot reflect the influence of different radius ratios and quantity ratios of the two sets of particles on the soil structure, packing style and shear strength. Therefore, this article focuses on analyzing the relationship between the absorbed suction of equal-sized particles with arbitrary packing and the influence of factors such as water content, dry density and contact angle.

To probe the relationship between the absorbed suction, water content, dry density and contact angle, it is assumed that the interfacial tensions $T_s = 72.7 \times 10^{-3}$ N/m, water density $\rho_w = 1.0$ g/cm^3, soil particle density $\rho_s = 2.7$ g/cm^3, the radius of soil particles $R = r = 10$ μm, the distance between particles $2d = 2$ μm and the contact angle $\theta = 10°$.

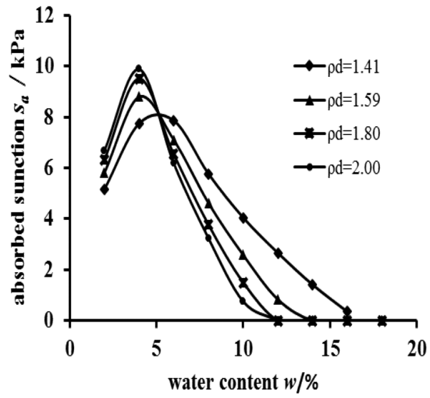


FIGURE 5. Absorbed suction and water content

The relationship between absorbed suction and water content

As it was shown in Figure 5, the absorbed suction increases first and then decreases until it reaches zero with increasing water content. With these variation characteristics, many strength behaviors of the unsaturated soil can be explained perfectly. For example, the strength of silt is low when it is very dry or bubbled in the water, but the strength increases first from a certain value (not zero because there is still structural suction, whose value is not zero though the absorbed suction is zero at this moment) then decreases until it reaches a constant value when its saturated degree changes from 0 to 1. The reason for this phenomenon is that the wetting effect of water strengthens the interactions between particles when the water content of the soil is low. However, with increasing water content, part of the effect of absorbed suction will decrease under the act of interfacial tensions and the interaction will weaken gradually until the effect of interfacial tensions acting on the particles becomes zero when the particles are totally wetted by the water. Therefore, although the effect of the structural suction should be considered when studying the strength behavior of soils, the variation trend of the above phenomenon is in accord with the (macroscopic) absorbed suction as the water content changes.

The relationship between absorbed suction and dry density

As it was shown in Figure 6, the absorbed suction increases with increasing dry density when the water content is low and the absorbed suction decreases with increasing dry density when the water content is high. The reason for this phenomenon is that when the water content is low, the particles in a unit volume will increase with an increase in the dry density and the contact points of one particle with other particles will increase. However, due to the limitations of particle packing (the maximum of coordination number is 12), the total action area of microscopic absorbed suction (equal to the value of the action area of microscopic absorbed suction of a single particle multiplied by half of the coordination number) is always less than the projection area of the hemisphere. Therefore, the (macroscopic) absorbed suction increases with increasing water content. When the water content is higher than a critical value located between 4 and 5%, the particles in a unit volume will increase with increasing dry density, causing the saturated angle and wetting area to increase. The total action area exceeds the projection area of the hemisphere, which causes the (macroscopic) absorbed suction to decrease. When the dry density is high, the water content will achieve its critical value in this density (the critical value of water content is 10.7% when the dry density is 2.0 g/cm^3) with a modest increase, but the (macroscopic) absorbed suction will rapidly decrease to zero.

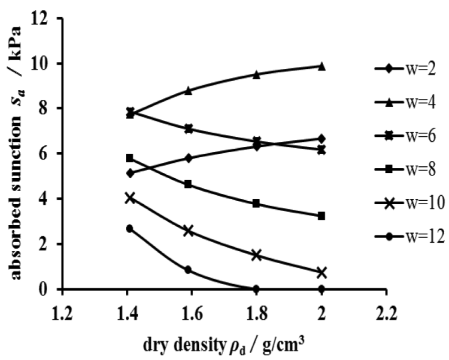


FIGURE 6. Absorbed suction and dry density

The relationship between absorbed suction and contact angle

As it was shown in Figure 7, the absorbed suction increases with increasing contact angle. As an example, when the water content is 4% and the contact angles are 0° and 30°, the relative difference of these two absorbed suction values can be calculated as $\left|\frac{11.38-7.06}{7.06}\right| = 90.9\%$. Obviously, the calculated results of absorbed suction will be influenced by the contact angle value. In fact, there are multiple types of mineral particles, such as quartz, mica and clay minerals, in the unsaturated soil (46) and the contact angles of unsaturated soil particles are different from 0° to 30°. Existing research has shown that the contact angle of quartz is 0 to 4°, mica is approximately 0°, montmorillonite is 139° (Diamond 1970) and the contact angle of others clay minerals is approximately 147° (Mitchell & Saga 2005). When the mineral grain percentage changes, the contact angle will experience a large change, thereby causing a significant change in the absorbed suction. This effect reflects the fact that the compositional characteristics of unsaturated soil have a great influence on the soil's mechanic properties (Yin et al. 2014, 2010).

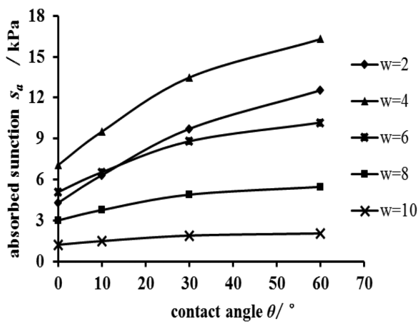


FIGURE 7. Absorbed suction and contact angle

QUANTITATIVE CALCULATION OF THE VARIABLE STRUCTURAL SUCTION

The derivation of a quantitative formula Based on the research findings of Zulkifly et al. (2014a, 2014b), the variable structural suction, which was proposed by Tang (2000a), can be computed with the following formula:

$$s_c = x \times s_0' \times J(S_r) = x \times s_0' \times (1 - S_r)^k, \tag{49}$$

where x is the number of contact points in a unit area which corresponds to the N mentioned in this paper; s_0 is the initial variable structural suction value of a single contact point; S_r is the degree of saturation which is related to water content and dry density; k is the structural parameter of soils which is constant for a certain soil and determined by the cement, the chemical component of pore water and the packing of the particles.

Equation (49) shows that the variable structural suction is determined by the degree of saturation and number of contact points in a unit area when k and s_0 are constant for a certain soil. The degree of saturation is a derived parameter from the water-soil and density state, which corresponds to the ratio of water volume to pore volume. The relationship between the degree of saturation, water content and dry density can be written as follows:

$$S_r = \frac{Dw\rho_d}{\rho_s - \rho_d}. \tag{50}$$

Meanwhile, the other influential factor x needs to be derived in different situations, as mentioned earlier and its values and formula is different for different packing conditions (Yoshitaka & Oka 2012).

Equal-sized unsaturated soil particles with arbitrary packing

Here, the number of contact points in a unit area, x, is written as follows:

$$x = N = \frac{\frac{1}{2}N_c N_s}{S}. \tag{51}$$

Substituting (15) and (24) into (51) yields the following:

$$x = \frac{26.493\rho_d - 10.727\rho_s}{4\pi R^2 \rho_d}. \tag{52}$$

By substituting (50) and (52) into (49), the variable structural suction can be written as follows:

$$s_c = \frac{26.493\rho_d - 10.727\rho_s}{4\pi R^2 \rho_d} \times s_0' \times (1 - \frac{Dw\rho_d}{\rho_s - \rho_d})^k. \tag{53}$$

The quantitative expression of the variable structural suction of equal-sized unsaturated soil particles with arbitrary packing has been obtained. Equation (53) shows that the variable structural suction is not only related to the soil density (soil particle density ρ_s, dry density ρ_d and relative density D) and water content but is also inversely proportional to the quadratic of particle radius, i.e. the variable structural suction will quickly increase with increases in the particle radius. This effect is why the structural strength of sandy soil is smaller than silt and clay.

Unequal-sized unsaturated soil particles with close tetrahedral packing

Here, with (48) for reference, the number of contact points in a unit area, x, is written as follows:

$$x = \frac{\frac{1}{2}N_c N}{S} = \frac{\rho_s}{4(\rho_s - \rho_d)(0.7R^2 + 0.3r^2)}. \tag{54}$$

By substituting (50) and (54) into (49), the variable structural suction can be written as follows:

$$s_c = \frac{\rho_s}{4(\rho_s - \rho_d)(0.7R^2 + 0.3r^2)} \times s_0' \times (1 - \frac{Dw\rho_d}{\rho_s - \rho_d})^k. \tag{55}$$

The quantitative expression of the variable structural suction of unequal-sized unsaturated soil particles with close tetrahedral packing has been obtained. Like the variable structural suction of equal-sized unsaturated soil particles with arbitrary packing, the variable structural suction in this case is also related to the soil density, water content and particle radius.

Discussion of the influencing factors of the variable structural suction The expressions of the variable structural suction under the combined conditions of different radii of the particles and packing patterns show that the variable structural suction is not only related to soil density and water content but also inversely proportional to the quadratic of the particle radius. Therefore, the degree of impact of the particle radius on the variable structural suction is larger than soil density and water content.

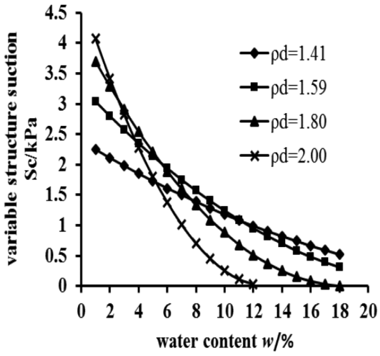
The parameters k and s'_0 are different for different soils and the values can be obtained via the corresponding related tests (Surhio et al. 2014; Willett et al. 2010). In order to probe the relationship between the variable structural suction, water content, dry density and the radius of soil particles, it is assumed that $s'_0 = 5.0 \times 10^{-10}$ kN, $k = 2$ [5] and the radius of soil particles $R = 10\ \mu m$ (in the case of equal-sized soil particles) or $R = 10\ \mu m$ and $r = 2\ \mu m$ (in the case of unequal-sized soil particles).

The relationship between variable structural suction and water content

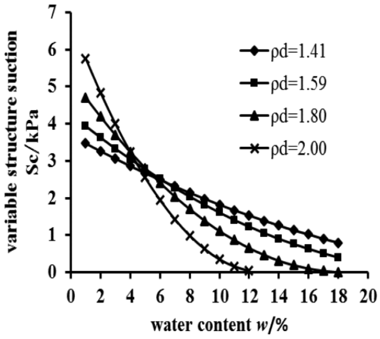
As it was shown in Figure 8, the variable structural suction of equal-sized unsaturated soil particles with arbitrary packing and unequal-sized unsaturated soil particles with close tetrahedral packing both decrease with increasing water content. By comparing the changes of the variable suction in the above two conditions, it is clear that the general trends are similar, but there are slight differences in the degree of reduction with the increasing water content. Assuming the same water content and density conditions, the variable structural suction of unequal-sized unsaturated soil particles with close tetrahedral packing is larger than that of equal-sized unsaturated soil particles with arbitrary packing. Furthermore, the range of the change of the latter is greater for a given interval of change in the water content. In other words, the variable structural suction of unequal-sized unsaturated soil particles with close tetrahedral packing is more influenced by the water content (Qureshi et al. 2015). The reason for this difference is that, for unequal-sized unsaturated soil particles, the pores of large particles will be filled with small particles, which results in closer packing of the soil particles and a greater number of contact points between one particle and those surrounding it. Eventually, the phenomena of cementation and occlusion between soil particles become more common. Furthermore, the latter is closely packed and the former is loosely packed, there are certain differences in the coordination numbers of the packing. Sometimes, the latter's coordination number may be even greater than 12, which never exists in the former case. Therefore, when the range of changes in the water content is the same, the latter experiences a greater change than the former, which is equivalent to magnifying the impact of water content on the variable structural suction.

The relationship between variable structural suction and dry density

As it was shown in Figure 9(a), the variable structural suction increases with increasing dry density when the water content is less than 2% and the variable structural



(a) Equal-sized unsaturated soil particles with arbitrary packing



(b) Unequal-sized unsaturated soil particles with close tetrahedral packing.

FIGURE 8. Variable structural suction and water content

suction decreases with increasing dry density when the water content is greater than 2%. In contrast, in Figure 9(b), the variable structural suction increases with increasing dry density when the water content is less than 3% and the variable structural suction decreases with increasing dry density when the water content is greater than 3%. The reasons for this phenomenon are that there are more particles in a unit volume of unequal-sized particles, which increases the degree of the soil wetted with water for a given water content. Hence, the impact of dry density on the variable structural suction in case of unequal-sized particles packing is more sensitive compared to that of equal-sized particles. In other words, when there is a small change in dry density, the variable structural suction will experience a large change, which makes it possible for the change range to skip the increasing then decreasing stage of suction with increasing dry density. Combining and comparing the changes of the variable structural suction in the above two packing patterns showed that the impact of dry density and water content on the variable structural suction are not independent, but related and connected with each other. This impact is like a 'weighted average' of the two effects and it is the result of their interaction. The distinction arises from the different 'weights' of two different states. The changes of the variable structural suction are different when the 'weights' of the two influence factors are different. This impact mechanism is identical to the impact of the water content and dry density on the macroscopic absorbed suction.

The relationship between variable structural suction and the radius of the soil particles

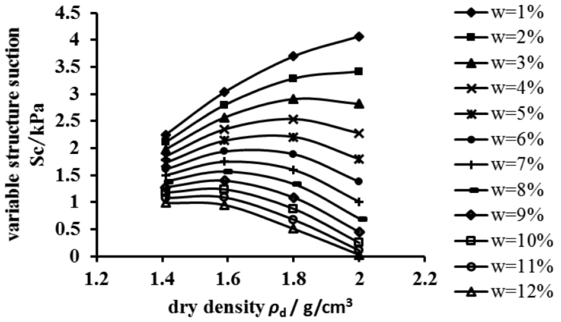
The impact mechanisms of the radius of particles on the variable structural suction of equal-sized unsaturated soil particles with arbitrary packing and unequal-sized unsaturated soil particles with close tetrahedral packing are the same. Therefore, in this paper, the variable structural suction of equal-sized unsaturated soil particles with arbitrary packing was used as an example to illustrate the impact mechanisms and rules of the particle radius on the variable structural suction. As shown in Figure 10, the variable structural suction drastically decreases with the increasing particle radius when the water content and dry density are constant. When the three variable structural suctions in the cases where the radius of the soil particles is 1 μm, 10 μm and 100 μm were compared, the last two are nearly negligible compared to the first. The degree of impact of the radius of the particles on the variable structural suction is much greater than that of the dry density and water content. The reason is because, with a decreasing radius, the specific surface area increases greatly, enhancing the surface activity of particles and resulting in a sharp increase in the cementation of particles. Furthermore, with a decrease in the radius of particles, the occlusion between the particles and the interaction between the molecules on the particle surfaces will also increase. The changes of various structural forces comprehensively lead to rapid increases of variable structural suction.

THE SHEAR STRENGTH THEORY BASED ON INTERGRANULAR SUCTION

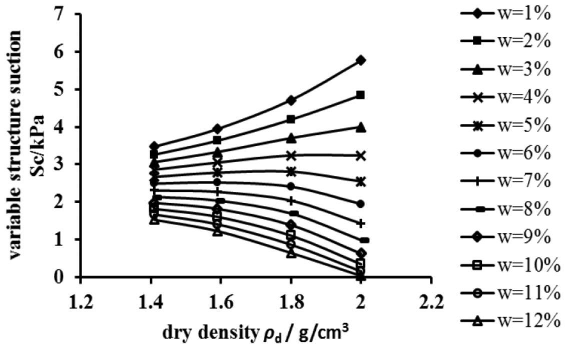
The external stress, σ, acting on a saturated soil is borne by the soil skeleton and pore water. The stress acting on the soil skeleton, causing the soil volume to change, is defined as the effective stress, σ'. The portion of stress acting on the pore water, not making the volume of soil change, is defined as the pore water pressure, u_w. The above relationship of the three stresses in saturated soil can be written as follows:

$$\sigma = \sigma' + u_w. \tag{56}$$

This formula is the principle of effective stress of saturated soil as put forward by Terzaghi (1943). From it, the nature of the principle of effective stress is easy to understand: The effective stress is delivered via the contact points between the soil particles that constitute the soil skeleton and the stress affects the soil's deformation and



(a) Equal-sized unsaturated soil particles with arbitrary packing



(b) Unequal-sized unsaturated soil particles with close tetrahedral packing

FIGURE 9. Variable structural suction and dry density

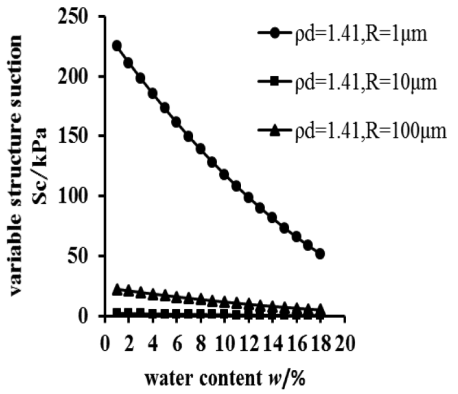


FIGURE 10. Variable structural suction and the radius of the soil particles (for equal-sized unsaturated soil particles with arbitrary packing)

strength, whereas the pore water pressure is just a neutral stress.

Through the comprehensive analysis of the interacting forces between particles, the intergranular suction consists of absorbed suction and structural suction. The absorbed suction is caused by the interfacial tensions between particles wetted by pore water, which is transferred via the contractile skin and draws particles closer to each other. The strength of the soil is strengthened by the existence of absorbed suction. Meanwhile, the structural suction is the general term related to the nature of the soil particle structure, which includes the bonding forces between particles, occluding force, electrostatic force with electric double layer, molecular force and magnetic force. Due to diversity, complexity and uncertainty of this suction, it was often selectively ignored in previous research. In fact, this suction is related to the structural strength of the soil, and its contributions to the soil strength cannot be ignored. Especially for fine-grained soils, the strength is greatly determined by the contribution of structural suction. Therefore, this factor cannot be ignored. For this reason, the theory of intergranular suction holds that absorbed suction and structural suction are an important part of effective stress, which conforms to the principle of effective stress. Considering that the intrinsic structural suction is a special type of variable structural suction, the structural suction can be generally called variable structural suction. Based on this, the effective stress formula of unsaturated soil was proposed by Tang (2000b):

$$\sigma' = (\sigma - u_a) + s = (\sigma - u_a) + (s_a + s_c). \tag{57}$$

This effective stress consists of two parts, where $(\sigma - u_a)$ is the soil's net stress, which is the additional effective stress caused by the external loads; and $(s_a + s_c)$ is the intergranular suction, where s_a is the absorbed suction; and s_c is the variable structural suction. Both s_a and s_c are the interaction forces between particles under the influence of solids and liquids, which directly act on the soil skeleton and are transferred via the soil skeleton.

Unsaturated soils are still subject to Mohr-Coulomb strength criterion, as proven by the existing experiments (Ashraf et al. 2013). Substituting (55) into the formula for Mohr-Coulomb strength, the shear strength formula of unsaturated soil based on the intergranular suction (including the absorbed suction s_a and the variable structural suction s_c) can be written as follows:

$$\begin{aligned}\tau &= c' + \sigma' \tan \phi' \\ &= c' + (\sigma - u_a) \tan \phi' + (s_a + s_c) \tan \phi'.\end{aligned} \tag{58}$$

As shown in (58), the shear strength of unsaturated soil consists of three parts: the effective cohesion; the soil's additional strength caused the external load; and the strength caused by the intergranular suction.

For the first part, the effective cohesive force, c', is constant when the unsaturated soil is under a certain condition, corresponding to the intercept of the shear strength envelope. The parameter c' will be changed when the soil moisture and/or density are changed. For sandy soil, c' is zero. For clay, the variation range of the shear strength index (c', φ') is larger and is related to the type of soil, the undisturbed structure of the soil, the stress history and the degree of consolidation. The variation range of φ is generally from 0° to 40°, whereas the variation range of c' is from several kPa to more than 200 kPa. The variation range of the schematic diagram is shown in Figure 11.

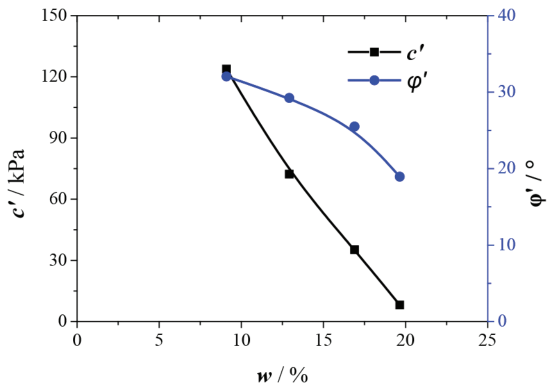


FIGURE 11. The variation range of the shear strength index (edited from Shen et al. 2009)

For the second part, the additional strength is caused by the external load, $(\sigma - u_a)\tan \varphi'$, where σ is the total normal stress of the shear slip surface and is easily obtained. In contrast, the pore air pressure, u_a, is related to not only the confining pressure but also the degree of saturation. For the same degree of saturation, the greater the confining pressure is, the greater the pore air pressure u_a. For the same confining pressure, the greater the degree of saturation is, the greater the pore air pressure u_a. For different combinations of the confining pressure and the degree of saturation, the pore air pressure u_a can vary from zero to 300 kPa and even larger. The variation ranges of the schematic diagram of the pore air pressure are shown in Figure 12.

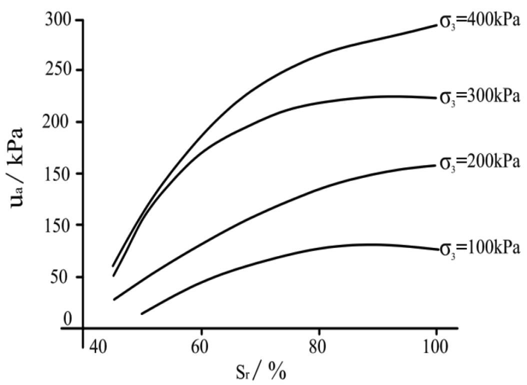


FIGURE 12. The variation range of the pore air pressure (Xing et al. 1996)

For the third part, the strength caused by the intergranular suction, $(s_a + s_c) \tan \varphi'$, is related to the water content, dry density and the radius of particles. The impact of the radius of particles is the most significant according to the formula derived in this paper.

Combining the above analyses, we know the following: For unsaturated sand (in which particles with radii larger than 0.075 mm account for more than 50% of the total mass), the strength of the first part is zero, the intergranular suction of the third part is less than 3 kPa and the additional strength of the second part ranges from zero to hundreds or even thousands of kPa. Thus, the contribution of the second part to the strength is far larger than the others, which should be considered more than the others because the others can almost be ignored; For unsaturated silt (in which particles with radii between 0.005 and 0.075 mm account for more than 50% of the total mass), the strength of the first part ranges from zero to 30 kPa, the intergranular suction of the third part ranges from zero to 27 kPa and the additional strength of the second part ranges from zero to 400 kPa or more depending on the imposed load. The contribution of the additional strength of the second part to the total strength is far larger than the others, but the contributions of the first part and the third part are auxiliary; and for unsaturated clay (in which particles with a radius of less than 0.005 mm account for more than 50% of the total mass), the first part ranges from zero to 200 kPa, the additional strength of the second part ranges from zero to 1000 kPa, and the intergranular suction of the third part ranges from zero to 400 kPa. Therefore, the contributions of the three parts to the total strength are important and the determination of the strength of soils should be considered comprehensively.

In conclusion, when the shear strength of unsaturated soil is studied, the impact factors, which include the imposed external load, the type of soil, the particle size, the confining pressure, the soil moisture states and the density states, must be considered comprehensively.

Conclusion

Based on the theory of intergranular suction, the general formulas for absorbed suction, variable structural suction and shear strength of unsaturated soil in different combinations of particle sizes and packing types are deduced. The relationships between the above quantities and their influencing factors, such as water content, dry density and the radius of particles were analyzed. The following conclusions were obtained:

Based on the particle model and the equilibrium conditions in the horizontal direction, the establishment of microscopic and macroscopic interconnected 'bridges' and the volume of pore water, we theoretically deduced the quantitative calculation formulas for absorbed suction of equal-sized and unequal-sized unsaturated soil particles with arbitrary packing and variable structural suction of equal-sized unsaturated soil particles with arbitrary packing and unequal-sized unsaturated soil particles with close tetrahedral packing. Their influencing factors were also discussed, which was a large step forward for the promotion of practical applications of the intergranular suction theory for unsaturated soils. The absorbed suction first increases and then decreases until it reaches zero with increasing water content. The absorbed suction increases with increasing dry density when the water content is low and the absorbed suction decreases with increasing dry density when the water content is high. The absorbed suction increases with increasing contact angle. The variable structural suction decreases with increasing water content. The variable structural suction increases with increasing dry density when the water content is low and the variable structural suction decreases with increasing dry density when the water content is high. The variable structural suction drastically decreases as the radius of particles increases. Among these three factors, the impact of the particle radius on the variable structural suction is much greater than those of dry density and water content. The shear strength of unsaturated soil consists of three parts: The effective cohesion, the soil's additional strength caused by the external load, and the strength caused by the intergranular suction. The contribution of the three parts to the total strength depends on the imposed external load, the type of soil, the size composition, the confining pressure, the soil moisture state and the density state. These factors should be considered comprehensively when determining the shear strength of unsaturated soil.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (Grant No. 40872205), the Specialized Research Fund for the Doctoral Program of Higher Education (No. 20120171110031), the Guangdong Natural Science Foundation (No. 07003738), and the Science and Technology Planning Project of Guangdong Province, China (No. 2008B030303009). The authors confirm that this article content has no conflicts of interest.

REFERENCES

Ashraf, M.A., Ullah, S., Ahmad, I., Qureshi, A.K., Balkhair, K.S. & Rehman, M.A. 2013. Green biocides, a promising technology: Current and future applications. *Journal of the Science of Food and Agriculture* 94(3): 388-403.

Baker, R. & Frydman, S. 2009. Unsaturated soil mechanics, critical review of physical foundations. *Engineering Geology* 106: 26-39.

Batool, S., Khalid, A., Chowdury, A.J.K., Sarfraz, M., Balkhair, K.S. & Ashraf, M.A. 2015. Impacts of azo dye on ammonium oxidation process and ammonia oxidizing soil bacteria. *RSC Advances* 5: 34812-34820.

Diamonds, S. 1970. Pore size distributions in clays. *Clays and Clay Minerals* 18: 723-724.

Gens, A. 2010. Soil-environment interactions in geotechnical engineering. *Geotechnique* 60: 3-74.

He, W., Zhao, M.H., Chen, Y.G. & Wang, H.H. 2010. Theoretical study of microscopical mechanisms and computational method of hysteresis in SWCCs. *Rock and Soil Mechanics* 31: 1078-1083.

Ibrahim, A., Mukhlisin, M. & Jaafar, O. 2014. Rainfall infiltration through unsaturated layered soil column. *Sains Malaysiana* 43(10): 1477-1484.

Jia, Q.J., Zhao, C.G. & Han, Z.D. 2005. Study on shear strength of the unsaturated soils in pendular state and its application. *Rock and Soil Mechanics* 26: 580-585.

Kim, T.H. & Hwang, C. 2003. Modeling of tensile strength on moist granular earth material at low water content. *Engineering Geology* 69: 233-244.

Lian, G.P., Thornton, C. & Adams, M.J. 1993. A theoretical study of the liquid bridge forces between two rigid spherical bodies. *Journal of Colloid Interface Science* 161: 138-147.

Lu, N. & Likos, W.J. 2006. Suction stress characteristic curve for unsaturated soils. *Journal of Geotechnical and Geoenviromental Engineering* 132: 131-142.

Luan, M.T., Li, S.Q. & Yang, Q. 2006. Matric suction and tension suction of unsaturated soils. *Chinese Journal of Geotechnical Engineering* 28: 863-868.

Meng, L.M. 2013. Tention atmosphere general formulas of effective stress principle of unsaturated soil. *The World of Building Materials* 34: 38-43.

Miao, T.D., Mu, Q.S., Liu, Z.Y. & Ma, C.W. 2001. Effective stress and shear strength of unsaturated soil with low water content. *Chinese Journal of Geotechnical Engineering* 23: 393-396.

Mitchell, J.K. & Soga, K. 2005. *Fundamentals of Soil Behavior*. 3rd ed. New Jersey: John Wiley & Sons, Inc.

Mu, Q.S., Ma, C.W. & Miao, T.D. 2004. Research on shear strength of unsaturated sand with low-moisture content. *Chinese Journal of Geotechnical Engineering* 26: 674-678.

Qureshi, T., Memon, N., Memon, S.Q. & Ashraf, M.A. 2015. Decontamination of ofloxacin: Optimization of removal process onto sawdust using response surface methodology. *Desalination and Water Treatment* 57(1): 221-229.

Ran, L.Z., Song, X.D. & Tang, C.S.D. 2011. Laboratorial investigation on tensile strength of expansive soil during drying. *Journal of Engineering Geology* 19: 620-625.

Shen, Z.J. 1996a. Retrospect and prospect of unsaturated soil mechanics. *Advances in Science and Technology of Water Resources* 161: 1-5.

Shen, Z.J. 1996b. Generalized suction and unified deformation theory for unsaturated soils. *Chinese Journal of Geotechnical Engineering* 18: 1-9.

Shen, C.N., Fang, X.W., Wang, H.W., Sun, S.G. & Guo, J.F. 2009. Research on effects of suction, water content and dry density on shear strength of remolded. unsaturated soils. *Rock and Soil Mechanics* 30: 1347-1351.

Shimada, K., Fujii, H.S., Nishiyama, T. & Morii, T. 2000. Change of shear strength due to interfacial tensions and matric suction of pore water in unsaturated sandy soil. *Proceedings of Unsaturated Soils for Asia*, edited by Balkema, A.A. pp.147-152.

Surhio, M.A., Talpur, F.N., Nizamani, S.M., Amin, F., Bong, C.W., Lee, C.W., Ashraf, M.A. & Shahid, M.R. 2014. Complete degradation of dimethyl phthalate by biochemical cooperation of the *Bacillus thuringiensis* strain isolated from cotton field soil. *RSC Advances* 4: 55960-55966.

Tang, L.S. 2000a. Structure suction and principle of general effective stress in unsaturated soils. *ACTA Scientiarum Naturalium Universitatis Sunyatseni* 39: 95-100.

Tang, L.S. 2000b. Mechanical effect of chemical action of water on soil and analysis on its mechanism. *ACTA Scientiarum Naturalium Universitatis Sunyatseni* 39: 104-109.

Tang, L.S. & Wang, S.J. 2006. Absorbed suction and principle of effective stress in unsaturated soils. *Chinese Journal of Geotechnical Engineering* 22: 83-88.

Tang, L.S., Sang, H.T., Song, J., Liu, F.T., Yan, B. & Zhang, P.C. 2013. Research on soil particle joint function and brittle-elastoplastic cement damage model of unsaturated granite residual soil. *Rock and Soil Mechanics* 34: 1453-1457.

Tang, L.S., Yan, B., Zhang, P.C. & Zhang, Q.H. 2006. Definition and exploration for effective stress and related conception in unsaturated soil. *Chinese Journal of Geotechnical Engineering* 28: 216-220.

Terzaghi, K. 1943. *Theoretical Soil Mechanics*. New York: Wiley Inc.

Wang, M. 1987. The interaction between water and clay particles in saturated clayey soils. *Hydrogeology and Engineering Geology* 13: 1-5.

Willet, C.D., Adams, M.J., Johnson, S.A. & Sevile, J.P.K. 2003. Effects of wetting hysteresis on pendular liquid bridges between rigid spheres. *Powder Technology* 130: 63-69.

Willett, C.D., Adams, M.J. & Johnson, S.A. 2000. Capillary bridges between two spherical particles. *Langmuir* 16: 9396-9405.

Xing, Y.C., Wu, P.A. & Luo, Y.S. 1996. Triaxial test on undisturbed unsaturated loess. *Journal of Hydraulic Engineering* 1: 47-52.

Yin, Z.Y., Zhao, J.D. & Hicher, P.Y. 2014. A micromechanics-based model for sand-silt mixtures. *International Journal of Solids and Structures* 51: 1350-1363.

Yin, Z.Y., Chang, C.S. & Hicher, P.Y. 2010. Micromechanical modelling for effect of inherent anisotropy on cyclic behavior of sand. *International Journal of Solids and Structures* 47: 1933-1951.

Yoshitaka, K. & Oka, C. 2012. Growth of liquid bridge in AFM. *Journal of Colloid and Interface Science* 40: 4368-4375.

Zhang, Z., Liu, F.Y. & Zhang, G.P. 2013. A physical hysteresis boundary water retention model for uniform wet particulate material. *Journal of Hydraulic Engineering* 44: 1165-1174.

Zhang, Z., Liu, F.Y., Zhang, G.P. & Zheng, F. 2007. Microscopic hydraulic behavior from the interactions between uneven-sized wet particles and liquid bridge. *Journal of Hydraulic Engineering* 28: 1323-1327.

Zhang, B.J., Yang, J.Q. & Ding, J.X. 2006. The effect of mineral component grain composition on CBR value. *Soil Engineering and Foundation* 20: 85-87.

Zhao, M.H., Liu, X.P. & Peng, W.X. 2007. Application of aqueous film theory to study of unsaturated soil's suction. *Rock and Soil Mechanics* 44: 810-817.

Zhao, C.G., Li, J., Liu, Y., Cai, G.Q. & Asreazad, S. 2013. Discussion on some fundamental problems in unsaturated soil mechanics. *Rock and Soil Mechanics* 34: 1825-1831.

Zulkifley, M.T.M., Ng, N.T., Abdullah, W.H., Raj, J.K., Ghani, A.A., Shuib, M.K. & Ashraf, M.A. 2014a. Geochemical characteristics of a tropical lowland peat dome in the Kota Samarahan-Asajaya area, West Sarawak, Malaysia. *Environmental Earth Sciences* 73(4): 1443-1458.

Zulkifley, M.T.M., Fatt, N.T., Raj, J.K., Hashim, R. & Ashraf, M.A. 2014b. The effects of lateral variation in vegetation and basin dome shape on a tropical lowland stabilization in the Kota Samarahan-Asajaya area, West Sarawak, Malaysia. *Acta Geologica Sinica* 88(3): 894-914.

Liansheng Tang*, Liqun Jiang & Yinlei Sun
School of Earth Sciences and Geological Engineering
Sun Yat-Sen University
Guangzhou, Guangdong 510275
China

Haitao Sang
School of Engineering
Sun Yat-Sen University
Guangzhou, Guangdong 510275
China

Haitao Sang
Guangdong Province Key Laboratory of Geological Processes and Mineral Resources Guangzhou, Guangdong 510275
China

*Corresponding author; email: eestls@mail.sysu.edu.cn

Received: 11 July 2014
Accepted: 15 November 2014