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The Application of Fractal Dimension on Capillary Pressure Curve to Evaluate the Tight Sandstone

(Aplikasi Dimensi Fraktal ke atas Lengkung Tekanan Kapilari untuk Menilai Batu Pasir Padat)

Chunhua Guo, Yu Yang*, Ping Yan, Ruili Zhou & Xiaodong Peng

ABSTRACT

The rock of gas tight reservoir is more heterogeneous than that of conventional sandstone reservoir and is more prone to water-blockage because of the invasion of operation fluid. This paper presented a new approach for the analysis of the capillary pressure curve for tight gas reservoir. Herein, the saturation equation with fractal dimension proved the previous observation that the log-log plot of capillary pressure against water saturation is a straight line, which is quite different from the popular observation by Corey's correlation. How to transform the capillary pressure curve to relative permeability curve was also discussed with fractal dimension. The fractal dimension of capillary pressure, which is not only an indication of heterogeneity, can also reveal the potential water blocks in tight gas wells. If the rock has higher fractal dimension, being at the same water saturation, the capillary pressure will be higher and the relative permeability of water will be smaller, which means higher injection pressure is required to displace the trapped water in reservoir. It is suggested that for the tight gas pay zone with higher fractal dimension, more precautions should be taken to prevent the water trapping during drilling or stimulating operation.

Keyword: Capillary pressure curve; relative permeability; tight sandstone; water trapping

ABSTRAK

Batuan di takungan kedap gas lebih heterogen daripada takungan batu pasir konvensional dan lebih cenderung kepada penahanan air kerana pencerobohan daripada cecair operasi. Kertas ini mengemukakan pendekatan baru untuk analisis lengkung tekanan kapilari takungan kedap gas. Oleh itu persamaan penepuan dengan dimensi fraktal membuktikan pemerhatian yang lepas bahawa plot log-log tekanan kapilari terhadap air penepuan adalah garis lurus dan agak berbeza daripada pemerhatian popular oleh Pekali Corey. Bagaimana mengubah lengkung tekanan kapilari untuk ketelusan bandingan lengkung juga dibincangkan dengan dimensi fraktal. Dimensi fraktal tekanan kapilari bukan hanya sebagai petunjuk kontra, juga boleh mendedahkan potensi blok air dalam telaga gas yang padat. Jika batuan mempunyai dimensi fraktal yang tinggi pada penepuan air yang sama, tekanan kapilari akan menjadi tinggi dan ketertelapan relatif air akan menjadi kecil yang bermaksud tekanan pancutan yang tinggi diperlukan untuk menggantikan air yang terperangkap dalam takungan. Adalah disarankan agar zon kaksa kedap gas dengan dimensi fraktal yang lebih tinggi memerlukan lebih banyak langkah keselamatan untuk mencegah air daripada terperangkap semasa penggerudian atau operasi perangsang.

Kata kunci: Batu pasir padat; ketertelapan relatif; lengkung tekanan kapilari; perangkap air

Introduction

It has been accepted by reservoir engineers that the heterogeneity of tight gas is more serious than conventional gas reservoir. How to characterize the microstructure of tight gas with the appropriate approach has been challenging for a long time. When studying the capillary pressure curve of tight gas, Wells and Amaefule (1985) found that the popular model of Brooks and Corey (1966) goes well with the sandstone with permeability greater than 10 mcrodarcy, but it may not be applicable in tight gas. Lekia and Evans (1988) observed that log-log plot of capillary pressure against water saturation, instead of normalized water saturation, is a straight line. With the introduction of fractal theory by Mandelbrot (1982), more and more researchers tried to utilize it to study the fractal nature of reservoir rocks and other porous media. It is reasonable to demonstrate the observation of Lekia and Evans (1988) with fractal theory.

Due to the complex pore structure, the relative permeability curve of tight gas reservoir is time-consuming to measure by the conventional stable or transient state method suitable for conventional reservoir and it is of great use to apply capillary pressure curve to compute the relative permeability curve.

Aqueous phase trapping represents a significant reason for impaired productivity in many gas reservoirs. Bennion et al. (1994) have also provided a detailed discussion of the mechanisms of aqueous phase trapping. Penny et al. (1983) pointed out that the capillary pressure, wettability and relative permeability are controlling the load water recovery following hydraulic fracturing treatments (Wang 2017). Since the fractal dimension has certain links with both of the capillary pressure and relative permeability, it can be used to evaluate the potential water blocking in tight gas reservoir.

THE CAPILLARY PRESSURE MODEL FOR TIGHT ROCK

Brooks and Corey (1966) made a large number of observation on consolidated rock cores and concluded that the capillary pressure P_c versus normalized wetting-phase saturation S^* could be expressed with the following relation,

$$P_c = C(S^*)^{-1/\lambda}. \tag{1}$$

Normalized wetting-phase saturation S^* is defined as:

$$S^* = \frac{S - S_{wi}}{1 - S_{wi}}, \tag{2}$$

where λ is the pore size distribution index; and C is a constant.

As indicated by (2), the P_c against S^* should be plotted as a straight line on log-log plot.

$$\lg P_c = \lg C - \lg(S^*)/\lambda. \tag{3}$$

However, Wells and Amaefule (1985) found that the model from Brooks and Corey (1966) goes well with the sandstone with permeability greater than 10 microdarcy and may not be applicable in tight gas. Lekia and Evans (1988) also observed that the Corey approach for the determination of the slope of P_c versus S^* is not applicable to tight gas sands. Based on the data from North-western Colorado, East Texas and Alberta and Canada, they found that the P_c against wetting phase saturation S on log-log plot is straight line.

Since both of the observations of Corey and Lekia were from the lab tests, why should there be the quite different conclusion? We can make a judgement from the capillary pressure model with fractal dimension.

In the 1980s, French mathematician Mandelbrot (1982) founded the new discipline of Fractal Geometry, which presents a simple but effective way to explain various complex natural phenomena. The matrix pore system in rock is fractal in every sense.

According to fractal theory, the fractal dimension of pore surface is given by

$$A \propto l^{2-D_a}, \tag{4}$$

where A is the area of pore surface; l is the length scale; D_a is the fractal dimension of pore surface, the value of which is between 1 and 2.

Yang et al. (2015) expressed the relation between the volume and radius of pore as;

$$V \propto r^{3-D}. \tag{5}$$

V is the volume of pore whose radius is r. D is the fractal dimension of pore volume, whose value is between 2 and 3.

Differentiating equation 5 gives:

$$\frac{dV}{dr} \propto r^{2-D}. \tag{6}$$

The cumulative volume of pores whose radii are greater than r is expressed as:

$$V(>r) = \int_r^{r_{\max}} ar^{2-D}dr = b\left(r_{\max}^{3-D} - r^{3-D}\right). \tag{7}$$

The total volume of connected pore is V_t expressed as:

$$V_t = \int_{r_{\min}}^{r_{\max}} ar^{2-D}dr = b(r_{\max}^{3-D} - r_{\min}^{3-D}). \tag{8}$$

The percentage of the cumulative pore volume whose radii are larger than r is expressed as S_m.

$$S_m = \frac{V(>r)}{V_t} = \frac{r_{\max}^{3-D} - r^{3-D}}{r_{\max}^{3-D} - r_{\min}^{3-D}}. \tag{9}$$

Because of the fact that $r_{\min}$ is much smaller than $r_{\max}$, (9) can be simplified as:

$$S_m = \frac{r_{\max}^{3-D} - r^{3-D}}{r_{\max}^{3-D}}. \tag{10}$$

In the earlier stage of mercury injection, the mercury is injected into the larger pores. During the injection of mercury, because the cumulative volume of mercury injected is the cumulative volume of pores whose radii are larger than r. S_m is also the saturation of non-wetting phase.

The Laplace equation is:

$$P_c = \frac{2\sigma\cos\theta}{r}. \tag{11}$$

Substituting (11) into equation (10), the relationship between P_c and the saturation of non-wetting phase S_m is:

$$S_m = 1 - \left(\frac{P_c}{P_{c\min}}\right)^{D-3}. \tag{12}$$

The saturation of wetting phase is defined as:

$$S = 1 - S_m = \left(\frac{P_c}{P_{c\min}}\right)^{D-3}. \tag{13}$$

Taking the logarithm of equation 13 on both sides yields:

$$\lg S = (D-3)(\lg P_c - \lg P_{cmin}) \tag{14}$$

The P_{cmin} is the entry capillary pressure.

It is suggested from (14) that the P_c against wetting phase saturation S on log-log plot is straight line, which confirms the conclusion of Lekia and Evans (1988).

To explain the different observations of Lekia and Corey, let's have a review of the cores used by them. The observation from Brooks and Corey are based on samples with higher permeability, whose irreducible wetting-phase saturation S_{wi} are much smaller than those of the tight samples Zhen and Gao (2017). That is to say, because the difference between S^* and S can be negligible for rock with small S_{wi}, the Corey's equation is reasonable for conventional gas pay zone with higher permeability. However, in the case of tight gas sample, the value of S_{wi} cannot be overlooked, the observation from Lekia is more reasonable. It should be stressed that the (14) is a general function between P_c and wetting phase saturation S, which can be used for all kinds of permeable rocks, especially the tight gas reservoir.

The relative permeability model

In 1949, Purcell recommended a method to calculate the relative permeability from the capillary pressure curve.

$$K_{rw} = \frac{\int_0^s ds/P_c^2}{\int_0^1 ds/P_c^2}. \tag{15}$$

$$K_{rnw} = \frac{\int_s^1 ds/P_c^2}{\int_0^1 ds/P_c^2}. \tag{16}$$

But the sum of K_{rw} and K_{rnw} by Purcell method, is 1 at any saturation, which does not coincide with the conclusion of the laboratory experiments that $K_{rw}+K_{rnw}<1$. Lekia and Evans (1988) also found a model for tight gas sand, it has been used successfully in the study of tight gas reservoir.

The gas and water relative permeability model given by Lekia and Evans is:

$$K_{rw} = s^2 \frac{\int_0^s ds\ /P_c^2}{\int_0^1 ds/P_c^2}. \tag{17}$$

$$K_{rg} = (1-s\ )^2 \frac{\int_s^1 ds\ /P_c^2}{\int_0^1 ds\ /P_c^2}. \tag{18}$$

Substituting the (13) into equation (17) and (18), the relative permeability are expressed as

$$K_{rw} = S^2 S^{\frac{5-D}{3-D}} = S^{\frac{11-3D}{3-D}}. \tag{19}$$

$$K_{rg} = (1-S\ )^2 (1-S^{\frac{5-D}{3-D}}). \tag{20}$$

Taking the logarithm of (19) on both sides yields:

$$\lg(K_{rw}) = \frac{11-3D}{3-D}\lg(S). \tag{21}$$

If we do not have relative permeability curve, we can compute the fractal dimension D from capillary curve with (14) and the relative permeability curve can be computed with (19) and (20).

The heterogeneity of tight rock samples with fractal

He and Hua (1998) calculated the pore size distribution of cores from oil reservoir and found that the fractal dimension represent the extent of heterogeneity Khairiah and Muhammad Syukri (2015). To demonstrate the relationship between heterogeneity and fractal, the geostatistical coefficients describing the heterogeneity of pore can be redefined with fractal dimension.

The mean value of pore radii $\bar{r}$ is defined as:

$$\bar{r} = \int_0^1 r ds_m. \tag{22}$$

Substituting (10) to (22) and rearranging it as:

$$\bar{r} = \frac{D-3}{r_{\max}^{3-D}} \int_{r_{\max}}^0 r^{3-D} dr = \frac{3-D}{4-D} r_{\max}. \tag{23}$$

The sorting coefficient of pore radii is defined as δ:

$$\delta = \left(\int_0^1 \left(r-\bar{r} \right)^2 ds_m \right)^{1/2}. \tag{24}$$

Substituting (10) and (23) into (24) gives:

$$\delta = \left[\frac{D-3}{r_{\max}^{3-D}} \int_{r_{\max}}^0 \left(r - \frac{3-D}{4-D} r_{\max} \right)^2 r^{2-D} dr \right]^{1/2} = \left[\frac{3-D}{5-D} - \left(\frac{3-D}{4-D} \right)^2 \right]^{\frac{1}{2}} r_{\max}. \tag{25}$$

The coefficient of variation C_v can also be given as:

$$C_v = \frac{\delta}{\bar{r}} = \left[\frac{(4-D)^2}{(3-D)(5-D)} - 1\right]^{1/2}. \tag{26}$$

TABLE 1. The geostatic parameters versus fractal dimension

D	$\bar{r}(\mu m)$	$\delta(\mu m)$	C_v
2.2	$0.444\, r_{\max}$	$0.297\, r_{\max}$	0.668
2.6	$0.286\, r_{\max}$	$0.292\, r_{\max}$	1.021
2.9	$0.091\, r_{\max}$	$0.198\, r_{\max}$	2.182

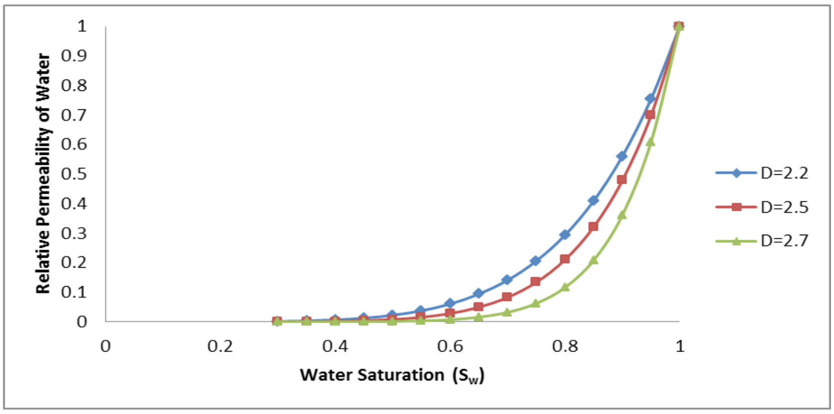

FIGURE 1. Relative permeability of water versus water saturation

Herein the fractal dimension is derived from the capillary pressure, which is the fractal dimension of pore distribution. Angulo and Ortiz (1992) found that the fractal dimension of pore volume scales is in the range 2.2 to 2.9. Krohn (1988) made a series of measurements on sandstone, shales and carbonates and found that the fractal dimension of sand stone is between 2.27 and 2.89. The fractal dimension of shales and carbonates are also with the same range.

Table 1 is computed with the capillary curves from tight sand stone samples of Peng Laizhen Group in Sichuan Basin. It is shown in Table 1 that if the fractals dimension increases, the coefficient of variation and the sorting coefficient will increase accordingly. This proves that the fact that fractal dimension can indicate the heterogeneity of pore distribution.

Predict the Potential of Water Trapping in Tight Gas Reservoir

The fractal dimension D can also be taken as a key parameter to evaluate the potential of water phase trapped in tight gas reservoir. The relative permeability of water is expressed as (19).

The shape of the water relative permeability indicates the potential of water trapping. It can be seen from Figure 1 that the concave curve shows a significant decrease of relative water permeability with a small decrease of the water saturation. If the fractal dimension D is 2.7, when the water saturation decreases from 1 to 0.8, the relative permeability of water will decrease from 1 to 0.1. However, if the fractal dimension D is 2.2, when the water saturation decreases from 1 to 0.8, the relative permeability of water will decrease from 1 to 0.3.

It is seen from Figure 1 that there is a lower relative permeability in the rock with higher D compared with that with smaller D at the same water saturation, which indicates that the trapped water will be much more difficult to be displaced by gas.

Conclusion

The capillary pressure model herein has demonstrated the observation of Lekia on tight gas that the capillary pressure against the water saturation in log-log plot is straight line. The model suggested by Corey that the capillary pressure against the normalized water saturation in log-log plot could be taken as a straight line is suitable for the rock with higher permeability, and cannot be used with tight pay zone. The fractal dimension can help to realize the conversion from the mercury-injection pressure to relative permeability curve, which is time consuming to measure by conventional experimental tests for tight rock sample. The fractal dimension, which is proved to be indication of heterogeneity, also indicates the potential water blocking. If the gas pay zone has higher fractal dimension, the relative permeability of water will be smaller at same water saturation. When drilling or stimulating the gas pay zone with higher fractal dimension, more measures should be taken in advance to prevent the water blocking.

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Chunhua Guo
College of Environment and Civil Engineering
Chengdu University of Technology
Chengdu 610059
China

Yu Yang* & Ping Yan
State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation
Chengdu University of Technology
Chengdu 610059
China

Ruili Zhou
Petro-Engineering Research Institute of North China Oil and Gas Branch
Sinopec, Zhengzhou 450006
China

Xiaodong Peng
Research Institute of Zhanjiang Branch
CNOOC Energy Technology & Services Co.
Zhanjiang 524057
China

*Corresponding author; email: yangyu@cdut.cn

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