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# On-line Detection Method for Outliers of Dynamic Instability Measurement Data in Geological Exploration Control Process

## (Kaedah Pengesanan atas Talian untuk Persilan Luar Pengukuran Data Ketidakstabilan Dinamik dalam Proses Penerokaan Kawalan Geologi)

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ABSTRACT

*Considering the characteristics of the vibration data detected by the unstable regulation process in the grinding and grading control system and the shortcomings of the traditional wavelet anomaly detection method, an online anomaly detection method combining autoregressive and wavelet analysis is proposed. By introducing the improved robust AR model, this method can overcome the problem that the time and frequency of traditional anomaly detection using wavelet analysis method cannot be well balanced and ensure the rationality of normal detection of process data. Considering the characteristics of parameter change and dynamic characteristics in the process of grinding and grading, the proposed method has the ability of on-line detection and parameter updating in real time, which ensures the control parameters of time-varying process control system. In order to avoid the problem that the traditional anomaly detection method needs to set the detection threshold, introduce the HMM to analyse the wavelet coefficients and update the HMM parameters online, which can ensure that the HMM can well reflect the distribution of the abnormal value of the process data. Through the experiment and application, it is proven that the anomaly data detection method proposed in this paper is more suitable for the detection data in the process of unstable regulation.*

*Keywords: Auto-regression; HMM; outlier detection; time series; wavelet*

ABSTRAK

*Dengan mengambil kira ciri data getaran yang dikesan melalui proses pengaturan yang tidak stabil dalam sistem kawalan pengisaran dan penggredan serta kelemahan kaedah pengesanan anomali tradisi gelombang kecil, kaedah pengesanan anomali atas talian yang menggabungkan autoregresi dan analisis gelombang kecil adalah dicadangkan. Dengan memperkenalkan model AR mantap diperbaik, kaedah ini boleh mengatasi masalah tidak boleh seimbangkan masa dan kekerapan anomali tradisi menggunakan kaedah analisis gelombang kecil dan memastikan rasionaliti pengesanan biasa dalam pemprosesan data. Dengan mengambil kira ciri perubahan parameter dan ciri dinamik dalam proses mengisar dan penggredan, kaedah yang dicadangkan mempunyai keupayaan pengesanan atas talian dan pengemaskinian parameter masa nyata dan memastikan parameter kawalan untuk sistem kawalan proses perubahan masa. Bagi mengelakkan masalah yang dihadapi oleh kaedah pengesanan anomali tradisi adalah perlu menetapkan tahap pengesanan dengan memperkenalkan HMM untuk menganalisis pekali gelombang kecil dan mengemaskini parameter HMM secara atas talian yang boleh memastikan bahawa HMM dapat menunjukkan pengagihan nilai data proses yang tidak normal dalam pemprosesan data. Melalui uji kaji dan aplikasinya, dibuktikan bahawa kaedah pengesanan anomali data yang dicadangkan dalam kertas ini adalah lebih sesuai untuk pengesanan data dalam proses peraturan yang tidak stabil.*

*Kata kunci: Auto-regresi; gelombang kecil; HMM; pengesanan pensilan luar; siri masa*

## INTRODUCTION

The grinding grade is one of the most important aspects in the process of mineral processing. The grinding and grading operation itself is a very complicated cycle process (Lindang et al. 2017; Lu 2016; Zhou et al. 2009). The interference factors are large, the process inertia is large, the pure lag is lagging, the nonlinearity is serious and the time is degenerate (Dai et al. 2015; le Roux et al. 2013). The grinding process control system through the detection of overflow particle size, according to the particle size model to adjust the amount of ore, water supply, to achieve particle size production indicators (le Roux & Craig 2013; Xu & Shin 2007). That is, in the production of the use of measured data obtained through the upper software system analysis and combined with the expert system to get the control of production conditions in line with the results. It can be seen, therefore, that the accuracy of the measured process data is critical for the grinding and grading control system.

Since the measured process data can reflect the real-time information characteristics of the grinding grading system, the definition of the anomaly cannot simply be equivalent to the traditional exception definition (away from most of the data (Grubbs 1969). In the grinding control system, the abnormal process data should be a kind of object which cannot reflect the characteristics of the actual grinding process information.

In this paper, we mainly analyze the data in the grinding process control system, which is one of the important aspects of mineral processing and consider the characteristics of the system with stable control structure and slow time-varying parameters and unstable adjustment process. The process data also has its own characteristics, for example, due to the large disturbance caused by the control process has a shock adjustment process, making the detection process data fluctuations and instability. In addition, due to the characteristics of time-varying system parameters make the mean or variance and other statistics also have time-varying characteristics, which require the detection algorithm has a high dynamic characteristics. Taking into account the large amount of process data, the detection algorithm requires on-line detection, the calculation is simple, you can detect the abnormal time in the required time. In summary, anomaly detection in the grinding process control system is a challenge.

For the study of anomaly detection methods, many good ideas and suggestions have been put forward, such as Barnett and Lewis in their book 'Outlier in Statistical Data' put forward based on statistical anomaly detection method (Barnet & Lewis 1994). Knorr and Ng proposed a distance-based anomaly data detection method (Knorr & Ng 1999, 1998). Ramaswamy et al. (2000) propose a new distance-based approach - density detection. However, due to the particularity of the abrasive process control anomaly definition, it is unreliable to judge the anomaly of the data only by the distance, density or mean and variance of the data. With the deepening of abnormal value detection technology, many new ideas and methods have been introduced. Such as clustering analysis (Seo 2016; Zhang et al. 2012), neural network (Bharti & Pattanaik 2016; Su et al. 2013) and support vector machine (SVM). The idea of clustering analysis is that the class with few samples is the abnormal class and all the data need to be categorized to make the abnormal judgment, thus, it cannot meet the requirement of online testing. For the neural network algorithm, which requires clean data offline training network and the calculation of complex, the same cannot be achieved by online testing which is the fast calculation of the requirement. SVM is very effective in detecting anomalies of small sample data, but the process data is magnanimous. If SVM is adopted, it will bring catastrophic computation. Therefore, the traditional methods are unable to meet the industrial regulation system generated by the abnormal time series of on-line detection requirements.

In mathematics, the abnormal data is defined as the amplitude or derivative of the function of the mutation, described by Lipschitz index, the smaller Lipschitz index, the greater the possibility of abnormal data (Othman et al. 2015; Pittner & Kamarthi 1999). Mallat and Hwang (1992) established the relationship between Lipschitz exponent and wavelet coefficient in 1992 and put forward the wavelet transform modulus maximum anomaly detection idea, which laid a solid mathematical foundation for wavelet anomaly detection. The wavelet has good adaptability to the signal, therefore, it can analyze the short-time high-frequency components in the signal, and estimate the trend of the low-frequency slowness. These characteristics of the wavelet are very applicable to the non-stationary signal analysis in the regulation system and process data feature extraction. Iterative wavelet analysis method (Rahman et al. 2017; Zhang et al. 1998) was proposed in 1998, which makes the wavelet analysis method can be carried out online and the calculation is simpler. Therefore, the signal analysis based on wavelet is widely used in various fields (Durocher et al. 2016; Griffiths et al. 2016; Han & Micheline 2001). However, since the wavelet method is a trade-off between time and frequency, there is no guarantee that time and frequency has good resolution at the same time. In addition, wavelet detection is based on the definition of abnormal values in mathematics, if the direct use of wavelet method to detect the abnormal process data, it seems unreasonable. Therefore, it is also inappropriate to simply use wavelet to detect the time series of the control process.

Considering the shortcomings of traditional anomaly detection methods and the characteristics of process data in grinding process control, this paper presents an anomaly data detection method based on autoregressive (AR) model and wavelet technology. The innovation of the method is described as follows:

Considering the characteristics of abnormal data definition of process data, the rationality of detection principle is guaranteed by using the AR model with simple structure and fast calculation speed, to fit the time series; In this paper, the proposed detection algorithm by using the wavelet technique to analyze the residuals of fitting and accurately detecting the outliers at high frequencies to overcome the problem of difficult to balance the time scale and frequency scale when do outliers detection by the traditional wavelet technique; The detection threshold is automatically set by using HMM to analyze the wavelet coefficients; and in this paper, we use the method of updating the parameters in real time to ensure that the detection algorithm has high dynamic characteristics and strong robustness. Considering that the anomaly value may affect the accuracy of the updated parameters, this paper minimizes the effect of outliers using the method of reducing the weight of the objective function by reducing the outliers.

The rest of this paper is organized as follows: we introduce the traditional wavelet anomaly detection method of the shortcomings by an example in the next section. Then, the AR-W-HMM anomaly detection method proposed in this paper for process data will be introduced in the following section. The simulation and validation of the proposed algorithm are presented next. Finally, the conclusions are summarized.

### THE EXISTENCE PROBLEMS OF TRADITIONAL WAVELET ANOMALY DETECTION

According to Mallat's modulus maxima theory, when the signal or its derivative is mutated, its Lipschitz exponent $\beta$ is less than zero, the relationship between the wavelet coefficients at a certain scale $s\,(s = 1/f)$ and the Lipschitz exponent $\beta$ is given in Mallat and Hwang (1992):

$$\left|W_f(s,t)\right| \le \left|Ks^{\beta}\right|, \tag{1}$$

wherein $K$ is a constant; and $W_f(s,t)$ denotes the wavelet decomposition coefficient under the scale $s$.

It is can be seen through (1) that when $\beta < 0$ and $s \Rightarrow 0$, $\left|W_f(s,t)\right|$ gradually decay, in other words, $\left|W_f(s,t)\right|$ is faster than the scale $s$ decay. Thus, at the position $\beta < 0$ representing a function or its derivative discontinuity, the wavelet transform value is formed as a modulus maxima. Traditional wavelet detection method is to find out the modulus maximum value of the way to detect abnormal data.

However, for the grinding process control system in the adjustment process, the data performance is very unstable, and there are cases of concussion data, but does not represent the abnormal value. The difference between them is that the oscillation process is continuous and is in the form of periodic fluctuations, and the outliers are spikes, but since both belong to the signal mutation. Therefore, the traditional wavelet detection method is very easy to normal oscillation adjustment signal mistaken for abnormal values. The above description is made in Figure 1 as an example.

Figure 1(a) shows the overflow concentration control curve in the grinding process control. We can see that at the initial stage (0-50 steps) curve shows oscillated which means that the data are oscillated, and then 15 fluctuations represent the anomaly data. Figure 1(b) is the result of the anomaly detection of the data of the overflow concentration control process by the traditional wavelet method. It can be seen from the test result that it will misinterpret the adjustment process as the abnormal value. Therefore, it can be seen from this example that the traditional wavelet detection method is not suitable for abnormal detection in the unstable adjustment process.

However, if the window size is appropriately adjusted on the basis of the example in Figure 1, the problem discussed can be alleviated. The result is shown in Figure 2.

In Figure 2, with the increase of the time window, the initial stage of the wave process no longer show the derivative of the 'sudden change', but moderate

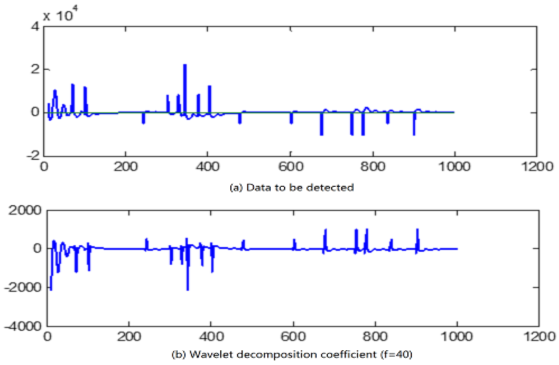


FIGURE 1. Detection example of traditional wavelet

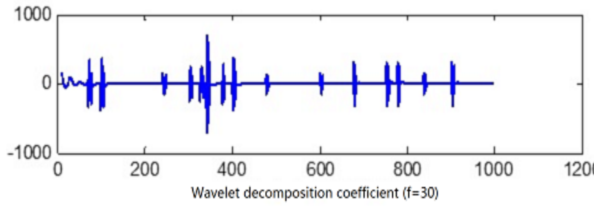


FIGURE 2. Wavelet decomposition under $f = 30$

fluctuations, thus no longer be mistakenly detected as an abnormal value. However, it can be seen from Figure 2 that the fluctuation range of the wavelet coefficients near the anomaly increases, indicating that the accuracy of the abnormal point positioning is reduced, resulting in more error detection. From the example, we can conclude that the traditional wavelet does not apply to the time series process of abnormal vibration detection tasks.

### AR-W-HMM ANOMALY DATA DETECTION ALGORITHM

Because of the shortcomings of traditional wavelet detection methods described in Figures 1 and 2, in this paper, an outlier detection method for using wavelet to analyze the residuals obtained from AR model is proposed, which method makes up for the shortcomings of the traditional wavelet method, making it suitable for adjusting the system time series anomaly detection problem.

In order to further improve the problem that the traditional wavelet method need to set the detection threshold in advance, this paper introduces the HMM to analyze the wavelet coefficients, thus avoiding the detection threshold setting problem. The specific process is shown in Figure 3.

## AR MODEL

### THE STRUCTURE OF AR MODEL

Takeuchi and Yamanishi (2006) used the AR model in 2006 as a fitting tool for time series outlier detection and according to AR model fitting residual Gaussian probability and pre-set detection threshold to determine

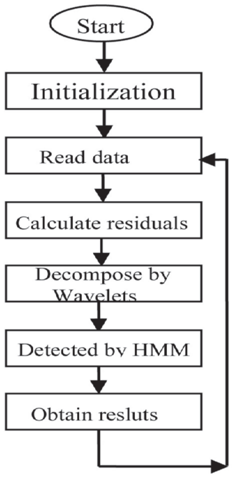


FIGURE 3. The flow of detection method

the data anomaly situation. Because AR model is simple in structure and fast in calculation speed, AR model is also used to fit the time series of process control system in this paper.

The data to be detected $x_t = [y_t, u_t^1, \cdots, u_t^{d-1}]^T$, $t = 1,2,\cdots$ is the $d \times 1$ dimensional column vector, where $y_t$ is the output of system at time $t$, and $u_t^j$, $j = 1,\cdots d-1$ is the input of system at time $t$. Using AR model to get the estimated value of the system sampling data at time $t$ (Takeuchi & Yamanishi 2006):

$$\hat{x}_t = \sum_{i=1}^{p} a_i x_{t-i} \quad , \tag{2}$$

where $\hat{x}_t$ is the fitted value of AR model to $x_t$, $a_i$, $i = 1,2,\cdots p$ is AR model coefficients and $p$ is the order of AR model.

From (2), we know that the system sampling value at time $t$ is related to the system sampling value before time $t$, which is in accordance with the actual situation of process control system. Therefore, the relationship between the input and output values of the system at time $t$ and the system input and output values before time $t$ can be approximated by the linear structure of (2).

The residuals $\varepsilon(t)$ obtained from the AR model are:

$$\varepsilon(t) = x_t - \hat{x}_t . \tag{3}$$

From (3), we know that if the data to be detected $x_t$ is normal data, $\varepsilon(t)$ is the white noise signal; if $x_t$ is abnormal data, then $\varepsilon(t)$ should be the sum of white noise signal and abnormal residual. In this paper, we consider that when the controlled object is disturbed by disturbances other than white noise, the generated data cannot represent the actual structure of the controlled object, so it is regarded as abnormal data. Therefore, there are two cases of residual sequence $\varepsilon(t)$, $t = 1,2,\cdots$: White noise or the sum of white noise and abnormal deviation.

However, for the regulation process, the input value of the controlled object $u_t^1,\cdots,u_t^{d-1}$ at time $t$ is the controller output, calculated by the computer, it is generally not unusual. So in this paper, $y_t$ in $x_t = [y_t, u_t^1, \cdots, u_t^{d-1}]^T$ is the data which need be detected. That is, in the subsequent wavelet analysis, only the fitting residuals of $y_t$ are analyzed.

## ROBUST TRAINING METHOD FOR AR MODEL

Due to slow time-varying characteristics of the controlled object parameters, an AR model with forgetting factor $r$ is used to train the AR model to ensure better dynamic performance.

But it is worth noting that the dynamic updating of the AR model on-line improves the dynamic characteristics of the model, but when the outliers appear, it is easy to update the AR model by using outliers to fit the outliers, which directly impact on the accuracy of detection of abnormal values, that is likely to cause missing phenomenon. In order to avoid the above problems, we add a weight item in the objective function to reduce the effect of outliers on the updating of model parameters and improve the robustness of the algorithm.

Specific training method is based on the idea of maximum a posteriori, objective function is:

$$\max \quad \sum_{i=1}^{t} r^{t-i} \cdot b_t \cdot \log p(\hat{x}_i \,|\, x^{i-1}, a_i) \quad , \qquad b_t = \begin{cases} 1, & y_t \textit{ is normal} \\ b_{t1}, & y_t \textit{ is outlier} \end{cases} \tag{4}$$

where, $b_t$ is the weight, when the data is normal is equal to 1; data anomalies take probability value $b_{t1}$ which calculated by (16), $b_{t1}$ represents the normal Gaussian probability of the data at time $t$. It can be seen that if the data is abnormal, then $b_{t1}$ will be very small, introducing it into objective function will reduce the impact of abnormal value on the accuracy of parameter updates. Therefore, as to achieve a robust effect, the concrete updating algorithm adopts the maximum likelihood estimation method and the calculation steps are as follows:

initialize parameters $a_1,\cdots a_p, C^0{}_j, D^0{}_j$ and read into the data $x_1,\cdots,x_p$;

for each reading data $x_t = [y_t, u_t^1, \cdots, u_t^{d-1}]^T$, $t = p+1, p+2, \cdots$, the following equation is calculated:

$$\begin{aligned} C^t{}_j &= (1-r)C^{t-1}{}_j + r \cdot b_t \cdot y_t x_{t-j}{}^T \quad (j = 1,\cdots p) \\ D^t{}_j &= (1-r)D^{t-1}{}_j + r \cdot b_t \cdot x_{t-1} x_{t-j}{}^T \quad (j = 1,\cdots,p) \end{aligned} \, , \tag{5}$$

where $r$ is the forgetting factor; $C^t{}_j$ is the $d$-dimensional row vector; and $D^t{}_j$ is the $d$-dimensional square matrix.

The AR model coefficients $a_i$ are obtained by solving equations.

$$C^t{}_j = \sum_{i=1}^{p} a_i D^t{}_{j-i} \quad (j = 1,\cdots p) \ , \tag{6}$$

where $a_i$ is the $d$-dimensional row vector, $D'_{-i} = D'^{T}_{i}$.

Since the outlier detection algorithm only detects the output value $y_t$ of the system, the AR model coefficients for fitting the value $y_t$ is needed only too in order to calculate the AR model coefficients in a simple and fast way.

## Improved recursive wavelet algorithm

In order to ensure the on-line detection of process data, an improved recursive wavelet algorithm (IRWT) (Zhang et al. 1998) which can realize on-line wavelet decomposition is used to decompose the fitting residual of AR model in this paper.

Define functions:

$$\Psi_1(t) = (\frac{\sigma^3 t^3}{3} - \frac{\sigma^4 t^4}{6} + \frac{\sigma^5 t^5}{15})e^{(-\sigma+i\omega_0)t}u(t). \quad (7)$$

Let $\Psi(t) = \Psi_1^*(-t)$ be the fundamental, the fundamental is:

$$\Psi(t) = (-\frac{\sigma^3 t^3}{3} - \frac{\sigma^4 t^4}{6} - \frac{\sigma^5 t^5}{15})e^{(\sigma+i\omega_0)t}u(-t). \quad (8)$$

Here, * denotes the conjugate; $\sigma = 2\pi/\sqrt{3}$ if; $\omega_0 = 2\pi$, then $\Psi(0) = 0$ and the basic wavelet satisfies the admissibility condition. The residual signal x is decomposed and discretized according to the wavelet in (8):

$$\begin{aligned} W_{\varepsilon,\Psi}(f,kT) &= \sqrt{f}T\sum_{n=1}^{\infty}\varepsilon(nT)\Psi^*(f(nT-kT)) \\ &= \sqrt{f}T\sum_{n=1}^{\infty}\varepsilon(nT)\Psi_1(f(kT-nT)) \end{aligned}. \quad (9)$$

Equation 9 is converted into a convolution type:

$$W_{s,\Psi}(f,kT) = T\sqrt{f}(\varepsilon(nT) * \Psi_1(fnT)), \quad (10)$$

where $T$ is the sampling period; $k,n$ is the integer used to mark the sampling point; $f$ is the reciprocal of the scale $s$;

The Z transformation of (10) can transform the convolution form into the product form:

$$W_{\varepsilon,\Psi}(Z) = T\sqrt{f}(\varepsilon(Z)\cdot\Psi_1(Z)), \quad (11)$$

where $\Psi_1(Z)$ can be expressed as

$$\Psi_1(Z) = \frac{\delta_1 Z^{-1} + \delta_2 Z^{-2} + \delta_3 Z^{-3} + \delta_4 Z^{-4} + \delta_5 Z^{-5}}{\lambda_1 Z^{-1} + \lambda_2 Z^{-2} + \lambda_3 Z^{-3} + \lambda_4 Z^{-4} + \lambda_5 Z^{-5} + \lambda_6 Z^{-6}}, \quad (12)$$

where

$$c = e^{-fT(\sigma - i\omega_0)}$$

$$\delta_1 = [\frac{(\sigma f T)^3}{3} - \frac{(\sigma f T)^4}{6} + \frac{(\sigma f T)^5}{15}]\cdot c$$

$$\delta_2 = [\frac{(\sigma f T)^3}{3}\cdot 2 - \frac{(\sigma f T)^4}{3}\cdot 5 + \frac{(\sigma f T)^5}{15}\cdot 26]\cdot c^2$$

$$\delta_3 = [\frac{(\sigma f T)^3}{3}\cdot(-6) + \frac{(\sigma f T)^5}{5}\cdot 22]\cdot c^3$$

$$\delta_4 = [\frac{(\sigma f T)^3}{3}\cdot 2 + \frac{(\sigma f T)^4}{3}\cdot 5 + \frac{(\sigma f T)^5}{15}\cdot 26]\cdot c^4$$

$$\delta_5 = [\frac{(\sigma f T)^3}{3} + \frac{(\sigma f T)^4}{6} + \frac{(\sigma f T)^5}{15}]\cdot c^5$$

$$\lambda_1 = -6c, \quad \lambda_2 = 15c^2, \quad \lambda_3 = -20c^3,$$

$$\lambda_4 = 15c^4, \quad \lambda_5 = -6c^5, \quad \lambda_6 = c^6,$$

Equation 12 into (11) can be obtained:

$$\begin{aligned} &W_{\varepsilon,\Psi}(Z)(1 + \lambda_1 Z^{-1} + \lambda_2 Z^{-2} \\ &+ \lambda_3 Z^{-3} + \lambda_4 Z^{-4} + \lambda_5 Z^{-5} + \lambda_6 Z^{-6}) \\ &= \sqrt{f}T\varepsilon(Z)\cdot(\delta_1 Z^{-1} + \delta_2 Z^{-2} \\ &+ \delta_3 Z^{-3} + \delta_4 Z^{-4} + \delta_5 Z^{-5}) \end{aligned}. \quad (13)$$

The discrete form of Eq.13 is the formula to improve the recursive wavelet decomposition:

$$\begin{aligned} &W_{\varepsilon,\Psi}(kT,f) = \sqrt{f}T\{\delta_1\varepsilon[(k-1)T,f] \\ &+ \delta_2\varepsilon[(k-2)T,f] + \delta_3\varepsilon[(k-3)T,f] + \\ &\delta_4\varepsilon[(k-4)T,f] + \delta_5\varepsilon[(k-5)T,f]\} \\ &- \lambda_1 W_{\varepsilon,\Psi}[(k-1)T,f] - \lambda_2 W_{\varepsilon,\Psi}[(k-2)T,f] \\ &- \lambda_3 W_{\varepsilon,\Psi}[(k-3)T,f] - \lambda_4 W_{\varepsilon,\Psi}[(k-4)T,f] \\ &- \lambda_5 W_{\varepsilon,\Psi}[(k-5)T,f] - \lambda_6 W_{\varepsilon,\Psi}[(k-6)T,f] \end{aligned}. \quad (14)$$

From (14), it is known that the initial 6 wavelet coefficients can be calculated by (9) and then the current wavelet coefficients at time t can be obtained by the fitting residual signal of the first 5 moments and the wavelet coefficients of the first 6 moments, so as to realize on-line wavelet decomposition, in order to meet the online abnormal data detection requirements.

## Analysis and detection algorithm based on HMM

As the traditional abnormal value detection method generally need to set the detection threshold in advance, that when the detection score is higher than the detection threshold that the data anomaly, otherwise the data normal. However, the detection threshold value needs to be calculated according to the actual calculation of the score range, if the set detection threshold is not accurate, it will seriously affect the detection accuracy. In this paper, we introduce the method of HMM to analyze the wavelet coefficients, thus avoiding the setting of detection threshold.

Hidden Markov Model is a 1-order double stochastic process which mainly consists of two parts (Jeff 2006; Rabiner 1989): One is the Markov chain, which describes the state of the transition, by the initial state probability $\pi$ and state transition matrix $A=(a_{ij})_{N\times N}$ where $a_{ij}=P(S_t=j|S_{t-1}=i)$ , $i,j\in S_s$, and $S_s$ denotes the set of all states, $S_t$ denotes the state at time *t*; *N* denotes the total number of states of the model;

The initial state transition probability $\pi$ is actually the initial value of the state transition matrix *A*, need to set in advance (here 0.5), while the state transition matrix of the elements $A$ used online update method:

$$a_{01}=\frac{N(a_{01})}{N(a_{01}+a_{00})} \quad , \quad a_{11}=\frac{N(a_{11})}{N(a_{11}+a_{10})}, \tag{15}$$

$$a_{00}=1-a_{01} \quad , \quad a_{10}=1-a_{11}$$

where marked the subscript of $a_{01}$"1" indicates that the data normal; "0" that the data abnormal, that is $S_s=\{0,1\}$. So $a_{01}$ represents the probability of which situation is that data at the previous time is normal and at the next moment abnormal, and g represents the number of times this occurs. Since *A* is updated online, only the number of occurrences of this situation before *t* is counted. As $A$ on-line constantly updated, so will be more accurate reflection of the detected time series outliers distribution.

Another stochastic process of the HMM describes the statistical relationship between the state and the observed value which is described by the observation probability matrix $B=(b_{tk})_{1\times 2}$, $k=0,1$, it represents the probability that the observed value takes a certain value under a certain state $S_t$.

In the detection algorithm proposed in this paper, $b_{t1}$ in the probability matrix of the observation $B$ represents the probability of similarity between the wavelet coefficients $W_{\varepsilon,\Psi}(t,f)$ at time $t$ and the wavelet coefficients $W_{ave}$ of normal data ( $k=1$, that is:

$$\begin{aligned} b_{t1} &= P(W_{ave},W_{\varepsilon,\Psi}(t,f)|S_t=1) \\ &= N(W_{\varepsilon,\Psi}(t,f)|W_{ave},W_{\text{var}}) \\ &= \exp(-\frac{1}{2}(W_{\varepsilon,\Psi}(t,f)-W_{ave})^T \\ &\quad W_{\text{var}}{}^{-1}(W_{\varepsilon,\Psi}(t,f)-W_{ave})) \end{aligned} \quad , \tag{16}$$

where $N(\cdot|\cdot)$ is the Gaussian similarity probability, since the residual signal is white noise when the time series of the wavelet analysis is the normal signal, $W_{ave}$ equal to the white noise mean 0.

In order to obtain the optimal state sequence of HMM $S_t$, $t=1,2,\cdots$ which represents the detection result of the data, in this paper, Viterbi algorithm (Lou 1995) is used. In Lou (1995), the real-time calculation method of Viterbi algorithm is given:

$$\begin{aligned} &\phi_t(1)=a_{i1}\cdot b_{t1}; \\ &\phi_t(0)=a_{i0}\cdot b_{t0}=a_{i0}\cdot(1-b_{t1}), \end{aligned} \tag{17}$$

where $\phi_t(1)$, $\phi_t(0)$ is the judgment index of $S_t=1$ (normal data); and $S_t=0$ (abnormal data) respectively; $a_{i1}$ represents the state transition probability of $i$ at the previous time; and 1 at the current time. Thus, the state chain value of the HMM which the result of detection at the current time can be determined simply by comparing the value of $\phi_t(1)$, $\phi_t(0)$ at each time:

$$\begin{aligned} &\phi_t(1)\ge\phi_t(0), S_t=1 \\ &\phi_t(1)<\phi_t(0), S_t=0. \end{aligned} \tag{18}$$

## Experiment and Application

### Experimental comparison

In order to verify the validity of the dynamic time series outlier detection algorithm proposed in this paper and to show that this algorithm is more suitable for time-varying and fluctuating time series outlier detection than traditional wavelet algorithm. We modified the model based on Alex et al. (2003) paper to obtain the experimental data. The modified model is as follows:

$$y(k)=\frac{y(k-1)y(k-2)y(k-3)u(k-2)(y(k-3)-1)+u(k-1)}{1+0.2\sin(2\pi k/25)+y(k-2)^2+y(k-3)^2}. \quad (19$$

Among:

$$u(k)=\begin{cases}\sin(\frac{2\pi k}{250}), & k\le 500 \\ 0.8\sin(\frac{2\pi k}{250})+0.2\sin(\frac{2\pi k}{25}), & k>500\end{cases} \tag{20}$$

As can be seen from (19), the denominator has time-varying parameter terms $0.2\sin(2\pi k/25)$. In order to verify the robustness of the algorithm, we add 10% white noise and eight outliers. The data and detection results are shown in Figure 5.

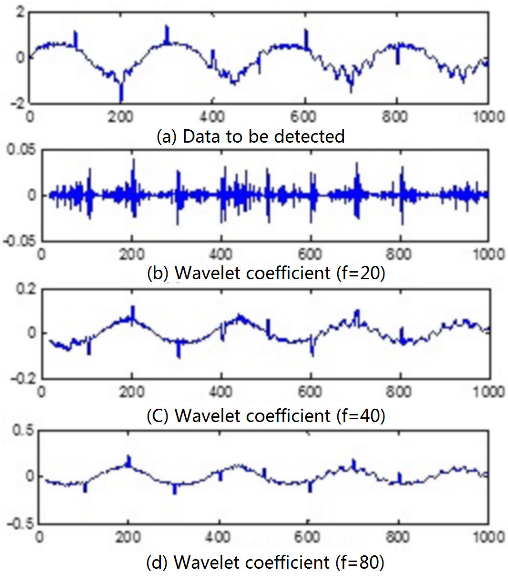


FIGURE 5. The signal from Alex's model and the detection results

As can be seen from the signal to be detected in Figure 5(a), its fluctuation is large and has time-varying characteristics. From Figure 5(b), it can be seen that the residual signal is almost white noise signal or white noise signal superposed with abnormal signal after fitting by AR model. As shown in Figure 5(c), the final detection result is obtained by wavelet analysis and HMM final optimization process (as shown in Figure 5(d)): All abnormal values are detected out accurately without misdetection and leakage.

Figure 6 shows the wavelet decomposition coefficient obtained by the wavelet analysis method without the AR model fitting processing. Since the signal fluctuation is more obvious here, the performance is more obvious that: Figure 6(b) shows the wavelet decomposition coefficients at small scales ( $f=20$ ) and it can be seen that there are many fluctuations around the anomaly, that is, it is difficult to locate the abnormal value accurately; While Figure 6(c) appropriately increases the scale value ( $f=40$ ), it can be seen that the fluctuation is similar to the original signal, therefore, it is difficult to determine the position of the outliers according to probability or HMM. Figure 6(d) shows the wavelet decomposition coefficients at the same scale as the wavelet decomposition in Figure 5 and the difference between the two decompositions on the same scale can be seen by comparing Figure 6(d) and Figure 5(c).

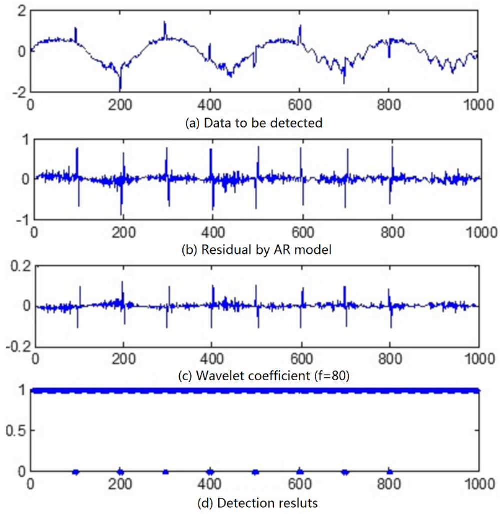


FIGURE 6. The signal from Alex's model and the analysis results based on traditional wavelet

The time taken to detect the two sets of data is shown in Table 1. As can be seen from the table, the average detection time is milliseconds, to meet the general regulation cycle.

It can be seen from the simulation experiments that the outlier detection method based the AR model and the wavelet analysis method proposed by this paper can solve the deficiency of the traditional wavelet analysis method. Therefore, the anomaly detection method based on improved wavelet analysis technology can be applied to the grinding process control system which is an kind of interference is more frequent in control system.

## Application

In order to further verify the practicability of the algorithm, this paper uses the process data (including 10 anomaly data) in the overflow concentration adjustment system of the grinding process control system to verify the results. The results are shown in Figure 7.

As shown in Figure 7(a), it can be seen that there is an oscillation adjustment process in the initial stage (1-100 steps), since the noise in the actual process data is larger than the noise signal in the simulation experiment. In addition, the amplitude of the outliers is small, therefore, there is a certain error in the fitted AR model at the initial stage, resulting in a larger fitting error at the initial stage (Figure 7(b)). In the detection results of Figure 7(d), a mistake (about 75 steps) occurred at the beginning because of the reason discussed earlier. Subsequently, as the model parameters are continuously updated, all the abnormal points are accurately detected.

TABLE 1. Time taken to detect two sets of data

| Number of data | Total detection time (s) | Average detection time (ms/per) |
|---|---|---|
| 1000 | 4.37 | 4.37 |

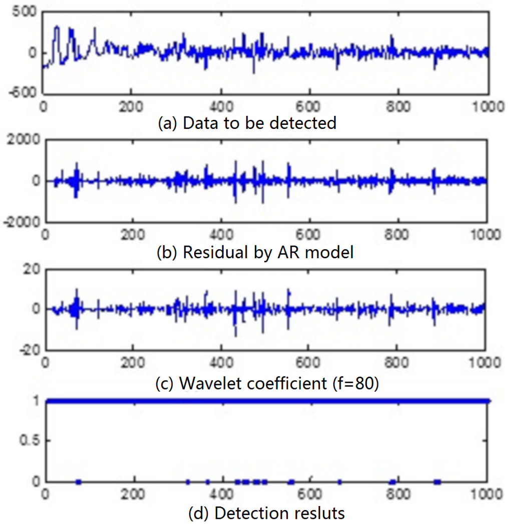


FIGURE 7. The real signal and detection results

## Conclusion

In this paper, a dynamic detection algorithm based on AR model and wavelet combination is proposed to detect the outliers of the time series of grinding control system. First, the AR model is used to linearly fit the data, which makes up for the shortcomings of using the wavelet analysis method to detect the lack of rationality of the anomalous data and the problem of not suitable for the fluctuation data. Then the residuals (white noise and possible anomaly signals) of the AR model fit are decomposed by wavelet at a certain scale. In order to realize the wavelet decomposition on-line, this paper uses the improved recursive wavelet decomposition algorithm to analyze the wavelet coefficients of certain frequency components. Finally, the HMM is used to describe the relationship between the wavelet coefficients and the data anomalies. The final detection result is determined by the Viterbi algorithm, which avoids the setting of the detection threshold and ensures the accuracy of the detection algorithm. Experiments and applications can prove that the proposed method for dynamic process control time series anomaly detection is more practical than using the wavelet method to detect the abnormal value of the vibration process data more accurately.


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