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Controlling the Blood Flow in the Stenosed Porous Artery with Magnetic Field

(Pengawalan Aliran Darah dalam Arteri Berliang yang Tersumbat dengan Medan Magnet)

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ABSTRACT

The unsteady blood flow in the stenosed porous artery subjected to a magnetic field was studied analytically. Oscillating pressure gradient and periodic body acceleration were imposed on the flow field. The effects of the magnetic field and the permeability of the stenosed artery on the blood velocity were studied. The results showed that the magnetic field affected the flow field significantly which can be beneficial for some practical problems.

Keywords: Magnetic field; porous medium; stenosed artery; unsteady blood flow

ABSTRAK

Aliran darah yang tidak stabil di dalam arteri berliang tersumbat yang dikenakan medan magnet dikaji secara beranalisis. Kecerunan tekanan berayun dan pecutan badan berkala telah dikenakan ke atas medan aliran. Kesan medan magnet dan arteri tersumbat pada halaju darah telah dikaji. Keputusan menunjukkan bahawa medan magnet mempengaruhi aliran dengan ketara yang bermanfaat untuk beberapa masalah sebenar.

Kata kunci: Aliran darah tak mantap; arteri stenosis; medan magnet; medium berliang

INTRODUCTION

The study of hemodynamics and hemorheology in constricted condition has recently gained the attention of many researchers. The formation of stenoses in arteries would alter the blood flow pattern in the circulatory system, thus causing numerous types of fatal cardiovascular. El-Shahded (2003) studied the pulsatile movement of blood flow through a permeable stenosed artery due to magnetic field. Makinde (2005) used analytical approach of perturbation method in solving the mathematical model describing the fluid dynamics of collapsible tube. It was observed that the model was appropriate to simulate wind tunnel tests on rheological phenomenon in physiological systems. Mustapha et al. (2010) investigated the blood flow through a couple of stenoses on an irregular surface. They showed that the pressure drop across the cosine stenosis exceeded the irregular one, which was consistent to the previous studies.

The use of magnetic field is common in treating stenosis. Sharma et al. (2015) examined the effect of magnetic field on the rheological models of blood. They highlighted that magnetic field could be used to control the blood flow, which was beneficial for certain hypertension cases. Tashtoush and Magableh (2008) studied the non-isothermal blood flow through multi-stenosed arteries subjected to magnetic field. Finite difference method was applied to solve the governing equations. Their results showed that the collision of particles within the arteries would increase the blood temperature and alter the blood flow pattern. Varshney et al. (2010) studied the blood flow through the stenosed artery in the presence of magnetic field. They found that the presence of multiple stenoses and magnetic field would affect the flow characteristics. Prakash and Makinde (2011) investigated the effect of radioactive heat absorption with the magnetic field resistance on the stenosed blood flow under the combination of pressure gradient and applied magnetic field. They reported that for patients undergoing thermal radiation therapy, the resistance to blood flow due to magnetic field and stenosis was reduced by increasing thermal radiation absorption. Singh et al. (2003) reported that the blood velocity, the flow rate and the shear stress would drop in the presence of magnetic field. The blood was modelled as the Herschel-Bulkley fluid. Xenos and Tzirtzilakis (2013) studied the stenosed blood flow in the presence of magnetic field as well. They showed that the velocity, the pressure and the skin friction were significantly affected upon applying the magnetic field. Bhatnagar and Shrivastav (2014) studied the blood flow in multiple stenosed arteries. Slip condition was considered and the flow field was subjected to a varying magnetic field. Bose and Banerjee (2015) reviewed the application of magnetic drug targeting (MDT) in treating stenosed aortic bifurcation. It is important to consider both ferrohydrodynamics (FHD) and magnetohydrodynamics (MHD) principles when the blood is modelled as a biomagnetic fluid. Shit and Majee (2015) studied the blood flow in an overlapping stenosed artery subjected to a magnetic field. The vibration of the whole body as well as the effect of heat were considered as well. Sharma et al. (2004) found that external magnetic field could decrease the velocities of blood and magnetic particles. Prakash et al. (2015) investigated the steady blood flow through the uniform cross-section blood vessels (modelled as Bigham plastic fluid) in stenosed arteries. It was observed that the stenoses size decreases the flow rate and increases the wall shear stress as well as resistance to flow. Based on the research performed by Siddiqui et al. (2015), wall slip would decrease the effective viscosity, the blood velocity and increase the flow resistance. Meanwhile, the shear wall stress decreased as the flow rate increased. Shah et al. (2016) showed that the MHD model yielded in different results if the fractional derivatives were employed. Majee and Shit (2017) investigated the unsteady non-isothermal blood flow through constricted arteries subjected to magnetic field. They highlighted that the artery wall would remain in the proper condition if the magnetic field strength was below 8T (beneficial for hyperthermic treatment).

The investigation of magnetic field within the porous medium is also important. Tanwar et al. (2014) investigated the effect of porous medium on blood flow in a stenosed artery subjected to a transverse magnetic field. As the permeability of the porous medium increased, the velocity decreased and the volumetric flow rate increased. Hatami et al. (2014) studied the non-Newtonian flow through a porous medium (magnetic hollow vessel) analytically and numerically. They reported that the velocity decreased as the MHD parameter was increased. Also due to the increase of the thermophoresis parameter (Nt), the temperature rising rate would be increased as well. Akbarzadeh (2016) showed that by lowering the Womersley parameter, the flow was dominated by viscous forces and the parabolic velocity profiles were obtained. Ullah et al. (2016) studied the velocity profile of the squeezed incompressible viscous flow through a porous medium under the influence of uniform transverse magnetic field. It was observed that both imposed magnetic field and electro conductivity would affect the fluid velocity. Bhatti and Abbas (2016) investigated the combined effects of slip and magnetic field on the peristaltic blood flow (modelled as Jeffrey fluid) through non-uniform porous channel. It was observed that the magnitude of the velocity is opposite near the walls due to slipping effects whereas similar behaviour has been observed for the magnetic field. Makinde et al. (2017) used Buongiorno's model in four different channel of the magneto-hemodynamic laminar flow namely, convergent, divergent, locally constricted and wavy channels. They showed that both thermal and Richardson numbers have opposite behaviour for skin friction, heat and mass transfer in each channel. Based on study conducted by Zaman et al. (2017) higher permeability could increase the blood flow rate (hence velocity) and the shear stress. They have considered the overlapping porous-saturated constricted artery subjected to body acceleration. The pulsatile blood (modelled as Herschel-Bulkley fluid) flow through a bifurcated artery was studied as well. The effects of periodic body acceleration and magnetic field were investigated (Ponalagusamy & Priyadhashini 2017).

In the current work, we extended the study of Akbarzadeh (2016) by controlling the blood flow in the stenosed porous artery via a magnetic field. The flow is treated as unsteady, laminar and incompressible. The analytical solution shall be presented by using the perturbation method and power series.

FORMULATION OF THE PROBLEM

The unsteady pulsatile laminar flow of incompressible Newtonian blood within a stenosed artery subjected to a magnetic field is considered (Figure 1). The stenosis is axially non-symmetric, however it is radially symmetric. The axial distance z and the height of its growth with the vessel radius (Siddiqui et al. 2015):

$$\frac{\bar{R}(\bar{z})}{\bar{R}_0}=\begin{cases}1-\bar{\psi}\left[\bar{L}_0^{m-1}(\bar{z}-\bar{d})-(\bar{z}-\bar{d})^m\right], & \bar{d}\le\bar{z}\le\bar{d}+\bar{L}_0\\ 1 & otherwise\end{cases} \quad (1)$$

where $\bar{R}(\bar{z})$ is the radius of the artery in the stenosed region; $\bar{R}_0$ is the radius of the normal artery; $\bar{L}_0$ is the length of the stenosis; $\bar{d}$ is the region of the stenosis; and $\psi=\frac{\bar{\xi}_s}{\bar{R}_0\bar{L}_0^m}\frac{m^{m/(m-1)}}{(m-1)}$ where $\bar{\xi}_s$ is the maximum height of the stenosis at $\bar{z}=(\bar{d}+\bar{L}_0)/m^{m/(m-1)}$ such that $\bar{\xi}_s/\bar{R}_0<1$. The blood is flowing in the z-direction through a fully porous vessel (or artery) of radius $\bar{R}(\bar{z})$ with an axial velocity of $u(z,r,t)$, i.e. the flow is assumed to be stable and axisymmetric with no radial and swirl components of velocity. It is supposed that, there is no-slip condition ($u=0$) on the outer wall ($r=R$). The problem involves the solution of both the Maxwell's relations for magnetic field and Navier–Stokes equations describing the blood flow.

The Maxwell equations are:

$$\nabla\cdot\vec{B}=0,\ \nabla\times\vec{B}=\mu_0\vec{J},\ \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial\vec{t}}, \quad (2)$$

where $\vec{B}$ is the magnetic flux intensity; μ_0 is the magnetic permeability; $\vec{E}$ is the electric field intensity; $\vec{t}$ is the time; and $\vec{J}$ is the current density given by,

$$\vec{J}=\sigma\,(\vec{E}+\vec{V}\times\vec{B}) \quad (3)$$

where σ is the electrical conductivity and $\vec{V}=(0,0,\bar{u})$ is the velocity distribution. For small magnetic Reynolds number, the linearized magneto-hydrodynamic force $\vec{J}\times\vec{B}$ can be expressed as

$$\vec{J}\times\vec{B}=\sigma\,(\vec{E}+\vec{V}\times\vec{B})\times\vec{B}=\sigma B_0^2\bar{u}, \quad (4)$$

where $\bar{u}$ (z, r, t) represents the axial velocity of the blood and B_0 is the externally applied constant magnetic field.

Based on the above considerations the equation of blood motion can be expressed as (Akbarzadeh 2016):

$$\rho\frac{\partial\bar{u}}{\partial\bar{t}}=-\frac{\partial\bar{p}}{\partial\bar{z}}+\mu\left(\frac{\partial^2\bar{u}}{\partial\bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial\bar{u}}{\partial\bar{r}}\right)+\bar{F}(\bar{t})$$

$$-\sigma B_0^2\bar{u}-\frac{\mu}{K}\bar{u},\ \bar{R}_0\le\bar{r}\le\bar{R}(\bar{z}), \quad (5)$$

where $\overline{p}$ is the pressure; $\overline{z}$ is the axial direction; $\overline{F}(\overline{t})$ is the body acceleration; ρ is the blood density; μ is the blood dynamic viscosity; σB_0^2 is the magnetic field strength; and K is the permeability of porous medium.

The blood flow was generated by the pressure gradient $\left(\frac{\partial \overline{p}}{\partial \overline{z}}\right)$ due to the pumping action of the heart (Akbarzadeh 2016; Siddiqui et al. 2015):

$$-\frac{\partial \overline{p}}{\partial \overline{z}}(\overline{z},\overline{t}) = A_0 + A_1 \cos(\overline{\omega}_p \overline{t}), \tag{6}$$

where A_0 is the steady-state part of the pressure gradient; A_1 is the amplitude of the pressure fluctuation giving rise to the systolic and diastolic pressures; $\overline{\omega}_p = 2\pi \overline{f}_p$ is the heart pressure frequency; and $\overline{f}_p$ is the pulse rate frequency.

The body acceleration $\overline{F}(\overline{t})$ can be expressed in its harmonic form (Akbarzadeh 2016; Siddiqui et al. 2015):

$$\overline{G}(\overline{t}) = A_g \cos(\overline{\omega}_g \overline{t} + \varphi), \tag{7}$$

where A_g is the amplitude of the acceleration; $\overline{\omega}_g = 2\pi \overline{f}_g$ is the frequency; $\overline{f}_g$ is the pulse rate frequency; and φ is the lead angle of the body acceleration with respect to the pressure gradient. Note that the effect of gravity in radial direction was neglected.

The boundary conditions on the velocity field are:

$$\overline{u} = 0, \quad \text{at} \quad \overline{r} = \overline{R}(\overline{z}), \tag{8}$$

$$\overline{u} \text{ is finite, at } \overline{r} = 0. \tag{9}$$

The non-dimensional governing equations can be obtained by introducing the following dimensionless variables:

$$R(z) = \frac{\overline{R}(\overline{z})}{\overline{R}_0},\ u = \frac{\overline{u}}{\frac{A_0 \overline{R}_0^2}{4\mu}},\ u_s = \frac{\overline{u}_s}{\frac{A_0 \overline{R}_0^2}{4\mu_f}},\ r = \frac{\overline{r}}{\overline{R}_0},\ t = \overline{t}\,\overline{\omega}_p,$$

$$\gamma = \frac{A_1}{A_0},\ B = \frac{A_g}{A_0}, \omega = \frac{\overline{\omega}_g}{\overline{\omega}_p}, \xi = \frac{\overline{\xi}_s}{\overline{R}_0}. \tag{10}$$

By using the above dimensionless variables, the following dimensionless problem can be obtained:

$$\alpha^2 \frac{\partial u}{\partial t} = f(t) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \left(Ha^2 + \frac{1}{K_1}\right)u, \tag{11}$$

subjected to the boundary conditions at any time t

$$\overline{u} = 0, \quad \text{at} \quad r = R(z), \tag{12}$$

$$\overline{u} \text{ is finite, at } r = 0 \tag{13}$$

where $\alpha^2 = \frac{\overline{\omega}_p \overline{R}_0^2 \rho}{\mu}$, α is the Womersley frequency parameter $Ha = \sqrt{\frac{\sigma}{\mu}} B_0 \overline{R}_0$ is the Hartmann number, $K_1 = \frac{\overline{R}_0^2}{K}$ is the porosity parameter and $f(t) = 4[(1 + \gamma \cos t) + B \cos(\omega t + \varphi)]$.

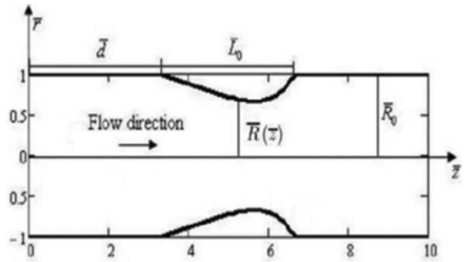

FIGURE 1. Geometry of stenosis

APPROXIMATE ANALYTICAL SOLUTIONS IN POWER SERIES FORMS

The main contribution of the current work is the development of the analytical solution of (12). Note that (12) could not be solved by applying merely the perturbation method for the expression of the blood velocity. Therefore, an approximate analytical solution is given in this section via combining the perturbation method and the power series. Assuming that the Womersley parameter is small ($\alpha^2 << 1$), where blood flows through small blood vessels such as arterioles and capillaries, its axial velocity component can be expressed in the following form:

$$u(z, r, t) = u_0(r, t) + \alpha^2 u_1(r, t) + \ldots \tag{14}$$

By substituting (14) into (11) – (13) we obtain,

$$\alpha^2\left(\frac{\partial u_0}{\partial t} + \alpha^2 \frac{\partial u_1}{\partial t}\right) = f(t) + \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial u_0}{\partial r} + \alpha^2 \frac{\partial u_1}{\partial r}\right)\right] - Q(u_0 + \alpha^2 u_1), \tag{15}$$

$$u_0 = u_1 = 0, \quad \text{at} \quad r = R(z), \tag{16}$$

$$u_0, u_1 \text{ are finite at } r = 0, \tag{17}$$

where $Q = Ha^2 + \frac{1}{K_1}$.

Simplifying (15),

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_0}{\partial r}\right) - (Qu_0 - f(t)) = 0, \tag{18}$$

and

$$\left(\frac{\partial u_0}{\partial t}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_1}{\partial r}\right) - Qu_1. \tag{19}$$

By considering (18) and manipulating the function

$$Qu_0 - f(t) = W_0(z, r, t), \tag{20}$$

both sides of (20) can be differentiated with respect to *r*, yields:

$$\frac{\partial u_0}{\partial r} = \frac{1}{Q}\frac{\partial W_0(z,r,t)}{\partial r} \tag{21}$$

Substituting (20) and (21) into (18), we obtain

$$r\frac{\partial^2 W_0}{\partial r^2} + \frac{\partial W_0}{\partial r} - QrW_0 = 0. \tag{22}$$

SOLUTION OF u_0 (z, r, t) IN THE FORM OF POWER SERIES

Assuming that the solution of (22) can be expressed in the form of power series:

$$W_0 = \sum_{n=0}^{\infty} C_n(z, t)r^n. \tag{23}$$

By substituting (23) into (22), the general terms for C_{2n+1} and C_{2n+2} can be obtained respectively as,

$$C_1 = 0 \tag{24}$$

$$C_{2n+1} = (2Q)^{2n}\left(\frac{n!}{(2n+1)!}\right)^2,\; C_1 = 0,\quad n = 0,1,2,\ldots, \tag{25}$$

$$C_{2n+2} = \left(\frac{Q}{2}\right)^{2(n+1)}\frac{C_0}{\left[(n+1)!\right]^2},\quad n = 0,1,2,\ldots, \tag{26}$$

Furthermore, from (24) - (26), (23) can be rewritten as:

$$W_0(z, r, t) = C_0 + \sum_{n=0}^{\infty} C_{2n+2}r^{2n+2}$$

$$= C_0 + C_0\sum_{n=0}^{\infty}\left(\frac{Q}{2}\right)^{2(n+1)}\frac{r^{2n+2}}{\left[(n+1)!\right]}, \tag{27}$$

Eq. (29) can be simplified further as,

$$W_0(z, r, t) = C_0(z, t)\, l_0(\sqrt{Q}r), \tag{28}$$

where l_0 is the modified Bessel function of order zero. Using (22), the solution u_0 can be expressed as

$$u_0 = \frac{W_0(z,r,t) + f(t)}{Q} \tag{29}$$

Applying the boundary condition $u_0 = 0$ at $r = R(z)$ we obtain

$$C_0(z, t) = -\frac{1}{2}\left[\frac{f(t)}{l_0\left(\sqrt{Q}R(z)\right)}\right]. \tag{30}$$

Therefore, the power series solution of u_0 can be expressed as

$$u_0 = \frac{f(t)}{Q}\left\{1 - \frac{1}{l_0\left(\sqrt{Q}R(z)\right)}\left[1 + \sum_{n=0}^{\infty}\left(\frac{\sqrt{Q}}{2}\right)^{2(n+1)}\frac{r^{2n+2}}{\left[(n+1)!\right]^2}\right]\right\} \tag{31}$$

SOLUTION OF $u_1(z, r, t)$ IN POWER SERIES FORMS

Substituting (31) into (20) and simplify to obtain:

$$r\frac{\partial^2 u_1}{\partial r^2} + \frac{\partial u_1}{\partial r} - Ha^2 u_1 r \tag{32}$$

$$= \left\{\frac{1}{Q}\frac{df(t)}{dt} - g(z,t)\left[1 + \sum_{n=0}^{\infty}\left(\frac{\sqrt{Q}}{2}\right)^{2(n+1)}\frac{r^{2n+2}}{\left[(n+1)!\right]^2}\right]\right\}r,$$

where

$$g(z,t) = \frac{1}{Ql_0\left(\sqrt{Q}R(z)\right)}\frac{df(t)}{dt}. \tag{33}$$

Considering the solution of u_1 as a power series in the form

$$u_1 = \sum_{n=0}^{\infty} D_n(z, t)r^n, \tag{34}$$

and substituting (34) into (32), yields,

$$(2k+2)^2 D_{2k+2} - QD_{2k} = \frac{g(z,t)}{(k!)^2}\left(\frac{\sqrt{Q}}{2}\right)^{2k},\; k = 1,2,3,\ldots \tag{35}$$

with the following recursive relation as:

$$D_2 = \frac{1}{4}\left[g(z, t) + QD_0\right], \tag{36}$$

$$D_{2n+2} = \frac{1}{(2n+1)}\left[QD_{2n-1} + \left(\frac{\sqrt{Q}}{2}\right)^{2n-1}\frac{1}{(n!)^2}\right], n = 1,2,3\ldots \tag{37}$$

$$D_{2n+2} = \left(\frac{\sqrt{Q}}{2}\right)^2\frac{D_0}{\left((n+1)!\right)^2}. \tag{38}$$

The power series solution of u_1 is expressed as:

$$u_1(z,r,t) = \frac{g(z,t)}{4}r^2 + D_0(z,t) + D_0(z,t)$$

$$\sum_{n=1}^{\infty}\left(\frac{\sqrt{Q}}{2}\right)^{2n+2}\frac{r^{2n+2}}{\left((n+1)!\right)^2} + \sum_{n=1}^{\infty} D_{2n+1}r^{2n+1}. \tag{39}$$

Using the boundary condition (16), we obtain the value as

$$D_0 = -\frac{\frac{g(z,t)}{4} R^2(z) + \sum_{n=1}^{\infty} D_{2n+1} R^{2n+1}(z)}{1 + \sum_{n=1}^{\infty} \left(\frac{\sqrt{Q}}{2}\right)^{2n+2} \frac{R^{2n+2}(z)}{\left((n+1)!\right)^2}} \tag{40}$$

The series solution is

$$u_1(z,r,t) = \frac{g(z,t)}{4} r^2 - \frac{\frac{g(z,t)}{4} r^2(z) + \sum_{n=1}^{\infty} D_{2n+1} R^{2n+1}(z)}{1 + \sum_{n=0}^{\infty} \left(\frac{\sqrt{Q}}{2}\right)^{2n+2} \frac{R^{2n+2}(z)}{\left((n+1)!\right)^2}}$$

$$-\frac{\frac{g(z,t)}{4} R^2(z) + \sum_{n=1}^{\infty} D_{2n+1} R^{2n+1}(z)}{1 + \sum_{n=0}^{\infty} \left(\frac{\sqrt{Q}}{2}\right)^{2n+2} \frac{R^{2n+2}(z)}{\left((n+1)!\right)^2}} \sum_{n=1}^{\infty} \left(\frac{\sqrt{Q}}{2}\right)^{2n+2} \frac{r^{2n+2}}{\left((n+1)!\right)^2}$$

$$+\frac{1}{(2n+1)}\left[QD_{2n-1} + \left(\frac{\sqrt{Q}}{2}\right)^{2n-1} \frac{1}{(n!)^2}\right]. \tag{41}$$

Finally, the perturbed power series solution for the velocity profile is:

$$u_0 = \frac{f(t)}{Q}\left\{1 - \frac{1}{I_0\left(\sqrt{Q}R(z)\right)}\left[1 + \sum_{n=0}^{\infty} \left(\frac{\sqrt{Q}}{2}\right)^{2(n+1)} \frac{r^{2n+2}}{\left[(n+1)!\right]^2}\right]\right\}$$

$$+\alpha^2 \left\{ \frac{g(z,t)}{4} r^2 - \frac{\frac{g(z,t)}{4} r^2(z) + \sum_{n=1}^{\infty} D_{2n+1} R^{2n+1}(z)}{1 + \sum_{n=0}^{\infty} \left(\frac{Q}{2}\right)^{2n+2} \frac{R^{2n+2}(z)}{\left((n+1)!\right)^2}} \right.$$

$$-\frac{\frac{g(z,t)}{4} R^2(z) + \sum_{n=1}^{\infty} D_{2n+1} R^{2n+1}(z)}{1 + \sum_{n=0}^{\infty} \left(\frac{\sqrt{Q}}{2}\right)^{2n+2} \frac{R^{2n+2}(z)}{\left((n+1)!\right)^2}} \sum_{n=1}^{\infty} \left(\frac{\sqrt{Q}}{2}\right)^{2n+2} \frac{r^{2n+2}}{\left((n+1)!\right)^2}$$

$$\left. +\frac{1}{(2k+1)}\left[QD_{2n-1} + \left(\frac{\sqrt{Q}}{2}\right)^{2n-1} \frac{1}{(n!)^2}\right]\right\}. \tag{42}$$

ANALYTICAL SOLUTION FOR LIMITING CASE

In this section, the analytical perturbation method was used to solve a special case of the model expressed in (15), by neglecting the magnetic field and the porosity of the medium (Q = 0):

$$\alpha^2 = \left(\frac{\partial u_0}{\partial t} + \alpha^2 \frac{\partial u_1}{\partial t}\right) = f(t) + \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial u_0}{\partial r} + \alpha^2 \frac{\partial u_1}{\partial r}\right)\right]. \tag{43}$$

Simplifying (43) results:

$$\frac{1}{r}\frac{\partial}{\partial r} r\left(\frac{\partial u_0}{\partial r}\right) + f(t) = 0, \tag{44}$$

and

$$\frac{\partial u_0}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial u_1}{\partial r}\right). \tag{45}$$

By integrating (44) and (45), using the boundary conditions, i.e. (12) and (13) we obtain:

$$u_0(z, r, t) = -\frac{1}{4} f(t)\, [r^2 - R(z)^2], \tag{46}$$

$$u_1(z, r, t) = -\frac{1}{64}\frac{df(t)}{dt}\, [r^4 - 4r^2R(z)^2 + 3R(z)^4] \tag{47}$$

Finally, the velocity profile will be as follows:

$$u(z, r, t) = -\frac{1}{4} f(t)\, [r^2 - R(z)^2] - \alpha^2\left\{\frac{1}{64}\frac{df(t)}{dt}[r^4 - 4r^2R(z)^2 + 3R(z)^4]\right\} \tag{48}$$

RESULTS AND DISCUSSION

In order to validate the current model, the results obtained by using the power series method and the perturbation method are compared in terms of physiological parameters selected based on Siddiqui et al. (2015). Figure 2 compares the current results with those obtained from (30) Siddiqui et al. (2015). Based on the result obtained, the blood motion equations in the absence of magnetic field and permeability are satisfied. Thus, the given solution was validated successfully. The blood velocity (hence flow rate) was higher in the absence of magnetic field.

Figures 3 - 4 show the time histories of the velocity profiles. Following Bose and Banerje (2015), the current

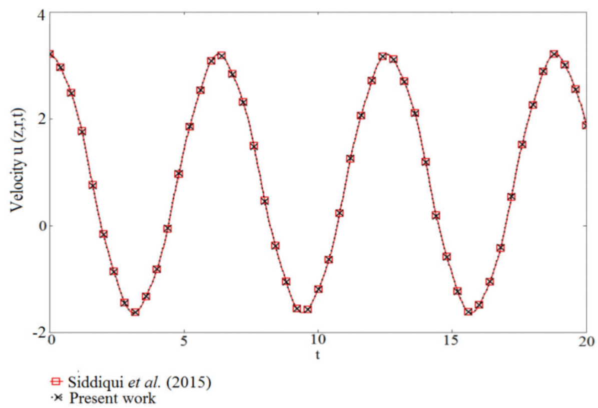

FIGURE 2. Comparison of velocity profile for blood distribution in the stenosed artery

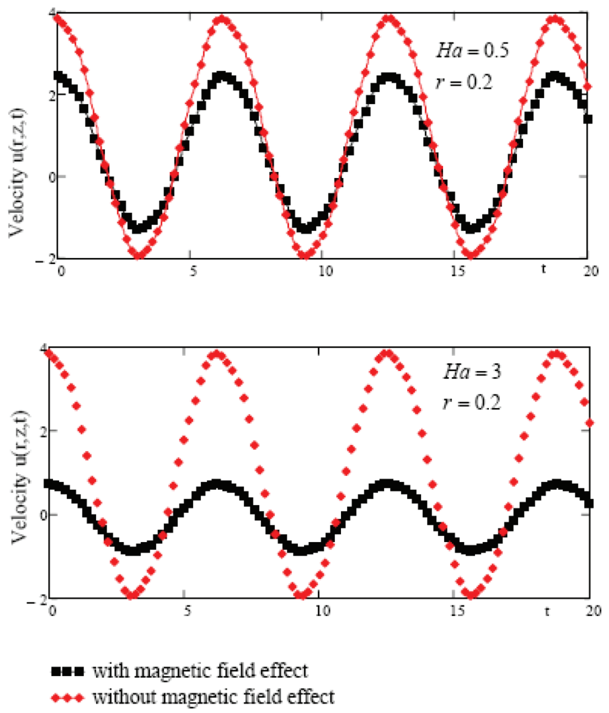

FIGURE 3. Velocity distribution of blood in the stenosed artery with/without magnetic effects for different values of Hartmann number Ha (when $K_1 = 0.2$, $r = 0.2$, $z = 1$)

dimensionless parameters were set as: $\alpha = 0.2$, $R_1 = 0.8$, $G=2, \frac{\bar{d}}{R_0}=\frac{10}{3}, \frac{\bar{L}_0}{R_0}=\frac{10}{3}, A=0.05, B=0.065, \frac{\xi_s}{R_0}=\frac{1}{3}, \omega=0.86,$ $\phi = 0.625$, $m = 10$. Figure 3 illustrates the influence of magnetic field on blood velocity. Apparently, the blood velocity decreased as the magnetic parameter in the boundary layer region of the artery increased (due to the Lorentz force). This force would decelerate the blood flow within boundary layer region.

Figure 4 illustrates the effects of permeability parameter K_1 on blood velocity with/without magnetic field. It is found from Figure 4 that blood velocity increases on increasing permeability parameter K_1 in the boundary layer region artery. This is because that an increase in K_1 results that there is decrease in the resistance of porous in the stenosed artery which tends to accelerate blood flow for magnetic field effects. Figure 5 shows the velocity profiles at different radial positions, *r*. Seemingly the blood velocity decreased as the radial distance increased. The flow resistance was lesser in bigger artery, thus decreasing the blood velocity.

Conclusion

In this research, we formulated a mathematical model describing the blood flow through stenosed porous artery with the application of magnetic field. The governing equations for both velocity are solved analytically and simulated on MATHCAD software. We found that the blood velocity decreases with an increasing of magnetic field, while increases with an increasing of permeability parameter. Besides the velocity profile also decreases as the size of radius increases.

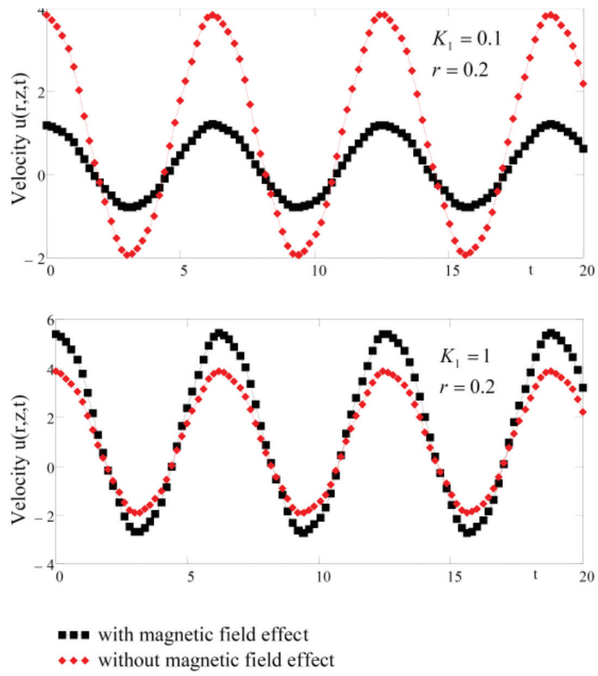

FIGURE 4. Velocity distribution of blood in the stenosed artery with/without magnetic effects for different values of permeability parameter K_1 (when $Ha = 1$, $r = 0.2$, $z = 1$)

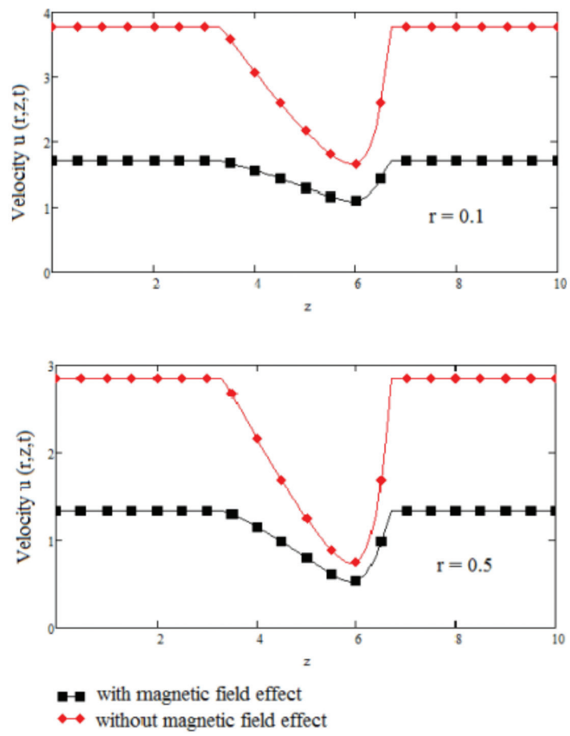

FIGURE 5. Velocity distribution of blood in the stenosed artery with/without magnetic effects with different value of radius (when $Ha = 1$, $r = 0.2$, $z = 1$)

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