Sains Malaysiana 48(7)(2019): 1547–1555
http://dx.doi.org/10.17576/jsm-2019-4807-24

Dependence Modeling and Portfolio Risk Estimation using GARCH-Copula Approach

(Pemodelan Kebersandaran dan Penganggaran Risiko Portfolio menggunakan Pendekatan GARCH-Copula)

RUZANNA AB RAZAK* & NORISZURA ISMAIL

ABSTRACT

Past studies have shown that linear correlation measure may result in misleading interpretations and implications of dependency when financial variables are involved. The copula approach can be adopted as an alternative for measuring dependence as it provides the solution to fat tail problems in multivariate cases which arises from the probability of large or extreme co-movements. Due to limited studies on copulas using Islamic financial data, this study set outs to obtain a clear picture on the dependence between Islamic and conventional stock markets in Malaysia. Firstly, we model the dependence between Islamic and conventional returns data using the copula-ARMA-GARCH models with normal and non-normal error distributions, and secondly, we evaluate the portfolios of Islamic and conventional indices using recent risk measures. This paper shows that, by using the copula approach for measuring the dependency between two financial variables while maintaining their true nature as described by the ARMA-GARCH models, meaningful interpretation can be made about the association of the financial variables which reflects the real association between markets. Furthermore, this study proposes a set of procedures on how portfolio risks can be estimated using VaR based on the ARMA(p,q)-GARCH(1,1)-t-copula models including backtesting via simulation.

Keywords: Copula; GARCH; risk; stock returns

ABSTRAK

Kajian lepas telah menunjukkan ukuran korelasi linear mungkin boleh menghasilkan interpretasi dan implikasi yang mengelirukan tentang kebersandaran yang melibatkan pemboleh ubah kewangan. Kaedah copula boleh diguna sebagai alternatif untuk mengukur kebersandaran. Kaedah ini memberikan penyelesaian kepada masalah ekor tebal dalam kes multivariat yang muncul daripada pergerakan bersama yang ekstrim. Kajian ini dijalankan untuk mendapatkan gambaran yang jelas berkenaan kebersandaran antara pasaran saham islam dengan konvensional di Malaysia kerana kajian berkenaan kaedah copula ke atas data kewangan islam adalah terhad. Pertama, pemodelan kebersandaran dilakukan menggunakan kaedah copula-ARMA-GARCH dengan taburan ralat normal dan tidak normal. Kedua, portfolio risiko dianggar melalui ukuran risiko yang terkini. Kajian ini menunjukkan kaedah copula boleh mengukur kebersandaran dan mengekalkan ciri sebenar pulangan saham yang diwakili oleh model ARMA-GARCH, di samping memberikan interpretasi bermakna yang mencerminkan hubungan sebenar antara pasaran saham. Selain itu, kajian ini mencadangkan prosedur penganggaran risiko menggunakan kaedah VaR yang berdasarkan model ARMA(p,q)-GARCH(1,1)-t-copula, termasuklah pengujian belakang melalui pendekatan simulasi.

Kata kunci: Copula; GARCH; pulangan saham; risiko

INTRODUCTION

The dependence between two variables of interest can be generally measured in several ways depending on the data. It is known that the linear correlation or Pearson correlation coefficient is inappropriate for financial returns since the returns are assumed to follow normal distribution; the stylized facts of financial returns suggest that the data deviates from the normal distribution. Therefore, correlation is not a good indicator of market interdependence.

Solutions to this problem have suggested the use of multivariate GARCH and copula approaches. The former is computationally exhaustive considering the number of models to be fitted, the number of parameters to be estimated and the diagnostic tests to be checked thoroughly. On the other hand, the copula approach offers a flexible method for modeling the return distribution of each stock market and the dependence between stock markets (Ning 2010). It provides the solution to fat tail problems in multivariate cases (or portfolios of multiple assets) which arises from the probability of large or extreme co-movements (tail dependence).

The suitability of modeling the dependence structures between financial variables using copula models have been recognized by previous studies on several types of dependence such as serial dependence, cross-dependence and cross-interdependence in stock markets. For example, Melo Mendes and Aíube (2011) used the copula approach to model the serial dependence in financial return series. Righi and Ceretta (2011) examined the overall dependence between the U.S. and Brazilian stock markets while Aloui et al. (2014), Hammoudeh et al. (2014) and Jäschke (2014) have looked at cross dependencies within the energy and international stock markets.

Risk, another aspect often studied especially at the wake of crises, can be measured in several ways depending on the investor's risk appetite. The standard deviation is the most widely used measure of investment risk which assumes that all investors agree on the degree of risk in every investment (Riddles 2001). However, investors are often concerned with the downside risk which measures the risk below some point. This downside risk can be measured using the value-at-risk (VaR) method. Despite its straightforward computation, VaR offers very limited guidance in exploring the tail function, particularly when it is known that the financial product has asymmetrical risk profile. Researchers have used the EVT-VaR and GARCH-VaR approaches as alternatives to obtain a more accurate measure of the downside risk and forecast risk, respectively. The recent developments in VaR methods have seen the use of copula-VaR in which the copula model serves as the parametric distribution of the data. The copula-VaR methods has been used for evaluating risk, such as in energy portfolios (Aloui et al. 2014; Jäschke 2014) and in conventional stock markets (Ab Razak & Ismail 2016). However, to our knowledge, this technique has yet to be applied in Islamic financial markets. Moreover, the risk evaluation of mixed weightage portfolios is very limited, at least in the perspective of the Malaysian stock market.

The main motivation for the strong interest in this study arises from the fact that there is limited published works on copula models that involve Islamic stock markets. This study set out to obtain a clear picture on the dependence between Islamic and conventional indices in Malaysia. This study also applies the value-at-risk methods, with the inclusion of copula, for evaluating the risk of naive and weighted portfolios which comprises of conventional and Islamic products. This study proposes a set of procedures on how portfolio risks can be estimated using VaR based on ARMA(p,q)-GARCH(1,1)-*t*-copula models including backtesting via simulation. Thus, the main objectives of this study were to model the dependence between Islamic and conventional indices in Malaysia for the period of years 2000 to 2012 using copula model with volatility models as marginal distributions, and to evaluate the portfolio risks of both indices using the recent risk measurement tool.

The structure of this paper is organized as follows. The first section provides the literature review of studies on the global and local stock market performances and the methods used for our investigation. Next, we describe how copula is used to model dependence and subsequently, in risk evaluation. After that, we present the findings and conclusion of this study.

Materials and Methods

Copula Modelling

The mathematical concept of copula was first introduced by Abe Sklar in 1959. Consider a vector of n variables. According to Sklar's theorem, a multivariate or joint distribution of the vector is a copula linking individual marginal distribution of each variable in that vector. The copula function can mathematically be expressed in terms of a joint distribution function, H, and the inverse of each marginal distribution in a vector (Nelson 2006). The copula function is:

$$C\left(u_1,u_2,...,u_n\right)=H\left(F_1^{-1}\left(u_1\right),F_2^{-1}\left(u_2\right),...,F_n^{-1}\left(u_n\right)\right) \quad (1)$$

where $F_1^{-1},F_2^{-1},...,F_n^{-1}$ are the quasi-inverses of marginal distribution functions for variables u_1, u_2, …, u_n, respectively.

The copula function provides the degree of dependence and dependence structure of multivariate cases. The copula approach has a number of attractive features. Firstly, copulas are invariant to transformations of data which is useful in finance studies where the original data is often transformed via appropriate tools such as log transformation (Ning 2010). Secondly, the scale-free measures of dependence (Nelson 2006) and the flexibility that it offers in modeling multivariate data (Melo Mendes & Aíube 2011) makes copula model interesting to statisticians since it allows users to separately model the marginal distribution of each variable and the dependence structure. Finally, the copula function can provide tail dependence index and captures the asymmetric dependence (Shamiri et al. 2011) which are often created from the fat tail problems in multivariate cases.

In bivariate context, the tail dependence measures the probability that the two variables are in the upper-right and lower-left quadrants which represent the extreme dependence of positive values and negative values, respectively. In finance, the lower tail dependence can be interpreted as the potential of simultaneous extreme losses in both stock markets where the joint loss is also known as the joint downside risk. The significance of the lower tail dependence implies that the two markets are correlated in times of crisis or bear market conditions, that is, they simply crash together.

On the other hand, the upper or right tail dependence measures the probability of simultaneous extreme positive values where the significance of the upper tail measure implies that both markets boom together. The dependence between paired indices in our study will be described by one of the copula families from the Archimedean, elliptical or extreme-value. In brief, the elliptical copulas (normal and student's *t*) have symmetric multivariate distribution, where the student's *t* copula has an advantage over the normal copula in terms of capturing the symmetric tail dependence. The Archimedean copulas, namely Clayton and Gumbel, have asymmetric distribution where each of them captures the dependence at the lower and upper tails of the multivariate distribution, respectively. Unlike the other Archimedean copulas, the Frank copula has a symmetric distribution and is relatively weak at capturing extreme dependence. Finally, the Galambos and Husler Reiss copulas belong to the extreme-value family where both copulas capture the upper tail dependence.

The estimation of dependence in our study uses the method of inference functions for margins (IFM) which consists of two stages. The first stage requires users to identify the marginal distributions of the data. Recent studies have used time series models such as DCC-GARCH (Righi & Ceretta 2011), EGARCH (Jäschke 2014; Shamiri et al. 2011), TGARCH (Hammoudeh et al. 2014) and ARMA-GARCH (Ab Razak & Ismail 2015; Aloui et al. 2014; Righi & Ceretta 2013) as marginal models. However, there is no consensus of which marginal model would influence the accuracy of the dependence estimation. Therefore, our study covers the standard GARCH models.

In the second stage, the residual series obtained from the marginal models are transformed into pseudo observations using U_i = Rank $X_i/(n+1)$ and V_i = Rank $Y_i/(n+1)$ where $i = 1,2,3,\ldots, n$. Given the pseudo observations, the Kendall's τ is computed and used for estimating the copula parameters. The Kendall's τ is a copula-based dependence measure that does not depend on marginal distributions (Heinen & Valdesogo 2012).

The goodness-of-fit (GOF) test is carried out to determine which copula model best describe the dependence structure of the paired stock markets. The null hypothesis for the GOF test is that the true copula, C, belongs to the assumed copula family. The test statistics for the GOF test uses the Cramer-von Mises statistics where its p-value of the test is computed using parametric bootstrap as described in Genest et al. (2009). The assumed copula model provides a good fit for the multivariate data when the p-value of the GOF test is greater than the significant level, say 5%.

MARGINAL MODELLING

The marginal models considered for each financial returns are statistical models that allow for autoregressive behavior, volatility clustering, skewness and fat tails (Stoyanov et al. 2011). The mean equation for each return series is modelled by an ARMA(p, q) process while the variance equation is modelled by a GARCH(P,Q) process. A good ARMA(p,q)-GARCH(P,Q) model at a specific p, q, P and Q lags should provide a parsimonious fit for each returns series and capture the asymmetric volatility clustering in the series, which can be checked by examining the model diagnostic tests. In general, the ARMA(p,q)-GARCH(P,Q) model with normal error distribution is expressed in the following equation:

$$
\begin{aligned}
Y_t &= \alpha_0 + \sum_{i=1}^{p} \varphi_i Y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t \\
\varepsilon_t &= X_t \sigma_t \quad \text{where } X_t \sim N(0,1) \\
\sigma_t^2 &= \omega + \sum_{i=1}^{Q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{P} \beta_j \sigma_{t-j}^2
\end{aligned}
\tag{2}
$$

where Y_t denotes the return of stock market index at the t–th day, σ_t^2 is the variance at the t–th day, ϕ_i and θ_j are the autoregressive and moving average coefficients, respectively, while $\alpha_i \varepsilon_{t-i}^2$ and $\beta_j \sigma_{t-j}^2$ are the ARCH and GARCH components, respectively. The residuals or error distribution, X_t, in equation (2) is assumed to follow normal distribution with zero mean and standard deviation of 1. Since GARCH processes are often heavy-tailed (Posedel 2005), the skewed normal, student-t with υ degrees of freedom, skewed student-t, generalized error and skewed generalized error distributions (see the work of Ghalanos (2015) for formulas of the error distributions) are also considered in our study for the distribution of X_t. The selection of the best ARMA-GARCH model that fits the returns series is based on diagnostic tests and model selection criterion.

The diagnostic checking involves observing the residual plots and its autocorrelation function (ACF) and partial autocorrelation function (PACF) diagrams, where the selected ARMA-GARCH model is appropriate if there are no significant lags in the diagrams. The Ljung-Box test of autocorrelation is applied to check the adequacy of the ARMA fit (Ghalanos 2015) by statistically examining the presence of serial correlation among the residuals. The test holds the null hypothesis of zero autocorrelation of the residual series. If the hypothesis is not rejected (or the p-value is greater than the significance level), we can conclude that no serial correlations are found in the residuals series. It is important to note that large sample sizes (or long series) might account for the significance of serial correlation at small lags (Ruppert 2011). To avoid biasness, the pattern of serial correlations at other lags are also observed such that they do not violate $l \leq 0.05n$. The ARMA-GARCH model needs higher-order modelling should the serial correlation remains in the residual series. The Lagrange Multiplier (LM) test is then employed to confirm the empirical adequacy of the marginal model. Finally, the model selection criterion (the formulas of the information criterion can be found in Ghalanos 2015) such as the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Shibata Information Criterion (SIC) and Hannan-Quinn Information Criterion (HQIC) are used to determine which marginal model that best fit the data series.

VALUE-AT-RISK (VAR) AND BACKTESTING PROCEDURES

This study compares the portfolio risk estimations generated from the normal VaR approach and the copula-VaR approach. The former approach provides the desired quantile of the normalized distribution of the portfolio returns. The copula-VaR approach involves simulating portfolio returns from individual distribution of each return of asset. Under this approach, the uniform variates for the two-asset portfolio are generated using the estimated copula. The variates are then transformed into standardized residuals. The returns series are then generated using both the standardized residuals and the dynamic terms observed in the actual return series. The portfolio VaR is then estimated using the desired quantile of the distribution of the portfolio returns.

The backtesting procedure is conducted to validate the efficiency of the copula-VaR approach in providing better risk estimates. The following steps were taken to perform backtesting via simulation:

Set the estimation window (*we*) such as 250 days and 1000 days; Set the confidence level; Estimate VaR for the first estimation window, $\text{VaR}_{(t=1)}$; Repeat step 3 to estimate VaR for the next window, $\text{VaR}_{(t=2)}$. Repeat this step until reaching the test window, $wt = W_T - we$, where W_T is the sample size; Record the number of violations by comparing the actual returns with the estimated VaR for every t, $t = 1,\ldots,wt$.; Calculate the expected number of violations using $(1 - \alpha)wt$., and Finally, calculate the violation ratio (VR) and perform the Kupiec's proportion of failures (POF) test.

A VaR model provides a good risk estimate if $VR \in [0.8, 1.2]$. The accuracy of the copula-VaR estimates of the portfolio is then ensured by applying the Kupiec's proportion of failures (POF) test where the null hypothesis is that the number of exceptions observed in a sample size T follows a binomial distribution with parameters T and α. The Kupiec's Likelihood Ratio test statistics complies with the chi-square distribution with 1 degree of freedom. The copula-VaR model provides a good estimate if the null hypothesis of the Kupiec's test is not rejected. It should be expected that the number of exceedances from the copula-VaR is closer to its expected number of exceedances.

Results and Discussion

The sample of daily data is extracted from Bloomberg, where the conventional stock markets are represented by the FTSE Bursa Malaysia Kuala Lumpur Composite Index (KLCI) and EMAS, while the Islamic stock markets are represented by the FTSE Bursa Malaysia Hijrah Shariah and EMAS Shariah. These indices represent the majority of stocks in Malaysia and are often used in studies of stock market performance.

Due to the non-stationarity of the price series, the log difference transformation was used to convert the series to returns series, and the characteristics of each returns series are empirically described in Table 1. The Ljung-Box Q-statistics was conducted to examine autocorrelation among the returns and squared returns, while The Jarque-Bera test was conducted to check if the returns are normally distributed or not. The sample correlation coefficient, which is measured by the nonparametric correlation method, is also provided to give an early insight about the association between the conventional and Islamic returns series.

The average return for Hijrah Shariah is higher compared to its conventional counterpart (KLCI), but the EMAS index has a slightly higher average return rate than its Islamic counterpart. Both Hijrah Shariah and EMAS have maximum returns compared to their respective counterparts, while both Shariah indices have the most negative returns. In terms of standard deviation, the KLCI has the lowest dispersion measure, suggesting that the KLCI returns are less volatile. The negative skewness proves that the distribution of each return series is non-normal, and the non-normality characteristics are further supported by the small *p*-values of the Jarque-Bera test for normality. Based on these results, both conventional and Islamic indices have many positive returns than negative returns. The values of kurtosis and excess kurtosis are quite large, indicating the existence of fat fails or extreme negative returns. The significance of Ljung-Box statistics provides

TABLE 1. Summary statistics of returns series

Statistics	KLCI	Hijrah Shariah	EMAS	EMAS Shariah
Mean	0.00022	0.00033	0.00019	0.00018
Median	0.00042	0.00042	0.00029	0.00040
Maximum	0.04503	0.04537	0.05127	0.04920
Minimum	-0.09979	-0.11090	-0.09949	-0.11320
SD	0.00891	0.00908	0.00935	0.00939
Skewness	-0.87021	-0.91182	-0.82806	-1.04026
Kurtosis	12.74277	14.18028	12.26231	14.99753
Excess Kurtosis	9.74277	11.18028	9.26231	11.997529
Q(12)	1.521e-14	2.475e-12	< 2.2e-16	1.18e-13
$Q^2(12)$	< 2.2e-16	< 2.2e-16	< 2.2e-16	< 2.2e-16
Jarque-Bera	< 2.2e-16	< 2.2e-16	< 2.2e-16	< 2.2e-16
Correlation	0.7168		0.8019	

Q(12) and $Q^2(12)$ denote the Ljung-Box test for returns and squared returns, respectively, at lag 12. Their values are *p*-values obtained from R software evidence of serial correlation within the returns and squared returns, suggesting that the returns of Islamic stock indices have similar statistical properties with the returns of conventional stock indices; i.e. leptokurtic distribution and extreme negative returns.

In terms of correlation, both sample correlation coefficients are significant at 99% confidence level, implying that the overall returns of conventional and Islamic indices are positive and strongly associated. Specifically, the EMAS and EMAS Shariah have a slightly stronger dependence than the KLCI and Hijrah Shariah. The positive and strong relationship indicates that the increasing or decreasing trend and the fluctuation of returns from Islamic indices are likely to follow similar patterns with the returns of conventional indices.

The line plots of return series are illustrated in Figure 1. It is apparent that volatility clusters are evident in 2000-2002 and 2007-2009, and each return series shows that the variance is not constant. By observing the two upper line plot in Figure 1, the returns of KLCI and Hijrah Shariah have similar movements in the overall sample period. The co-movement seemed stronger for the pair of EMAS and EMAS Shariah, confirming the positive and strong correlation value provided in Table 1. During crisis periods, the returns of EMAS and EMAS Shariah indices have almost identical return movements.

The series are modeled by the time series process. The ARMA specifications (autoregressive lags and moving average lags) are identified first through correlogram or autocorrelation function plots, and various ARMA models are fitted to each return series. The best fit ARMA model for each series is selected based on the smallest Akaike Information Criterion (AIC) value. Further statistical tests for diagnostic checking such as the Ljung-Box Q-statistics test for autocorrelation of residuals and the Jarque-Bera test for normality of residuals are applied to validate the fitted ARMA model. The results, which are not shown in this article, showed that the specified ARMA processes do not violate the invertibility and non-autocorrelation assumptions and the residuals are non-normally distributed. Using the Lagrange Multiplier (LM) test for ARCH effects, the statistical results showed the rejection of the hypothesis of no ARCH effects at 1% significance level. The ARCH(1) and ARMA(p, q)-ARCH(1) models, where p and q refer to the best fit ARMA (p, q) model, are incapable of estimating the conditional variance for all return series. Therefore, the ARMA(p, q)-GARCH(1,1) models with various error distributions are fitted to each series. The best fitted models from the ARMA(p, q)-GARCH(1,1) models are shown in Table 2.

The AR(1)-GARCH(1,1) with student's t distribution and the ARMA(2,2)-GARCH(1,1) with student's t distribution are selected to model the KLCI and EMAS index, respectively. For Islamic indices, the AR(2)-GARCH(1,1) with skewed student's t distribution and the ARMA(1,1)-GARCH(1,1) with skewed student's t distribution are chosen as the best model for the Hijrah Shariah and EMAS Shariah indices, respectively. The parameter estimates, denoted by ϕ_i and θ_i, are the i-th autoregressive and moving average coefficients, respectively, where the significance of these coefficients can be interpreted as today's return rate is affected by previous return rate and previous residual. The correct specification of the ARMA model is justified by the insignificant values of the Ljung-Box Q statistics of residuals and the lowest values of the information criterion. The most important coefficients of volatility models are α_1 and β_1, where the significance of α_1 indicates the presence of volatility cluster in the series, while ($\alpha_i + \beta_i$) measures the persistence effects of shocks on volatility. If $\alpha_i + \beta_i = 1$, the effects of shocks on volatility is said to be permanent; otherwise, the effects are transitory. For all series, the effects of shocks on volatility of returns are not permanent.

The Q-statistics for residuals at lag 10, 15 and 20 have small p-values, but they are insignificant at 1% level

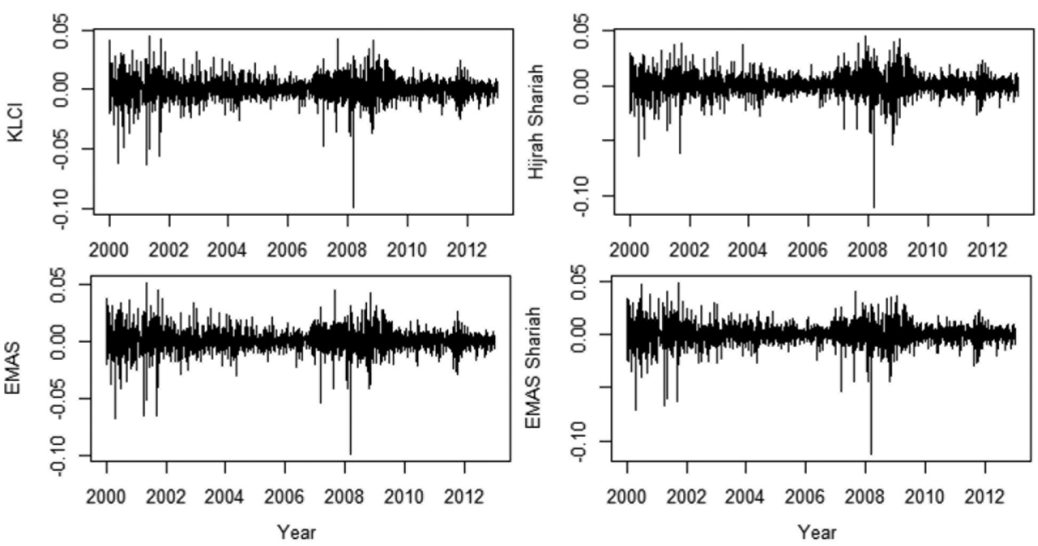

FIGURE 1. Line plots of conventional and Islamic stock market returns series

TABLE 2. Volatility models: Parameter estimates, diagnostic tests and information criterion

	KLCI	Hijrah Shariah	EMAS	EMAS Shariah
Parameter estimates				
μ	0.00040*	0.00041*	0.00037*	0.00021**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
ϕ_1	0.12992*	0.10982*	-0.04470	0.40519*
	(0.0179)	(0.0175)	(0.2227)	(0.1241)
ϕ_2	-	0.00947	0.23910***	-
		(0.01762)	(0.1239)	
θ_1	-	-	0.18028	-0.28337**
			(0.2231)	(0.1305)
θ_2	-	-	-0.19696***	-
			(0.1111)	
ω	1.07E-06*	8.91E-07*	1.01E-06*	9.38E-07*
	(3.14E-07)	(2.804E-07)	(2.90E-07)	(2.67E-07)
α_1	0.11473*	0.08350*	0.11461*	0.09866*
	(0.0169)	(1.16E-08)	(0.0161)	(0.0151)
β_1	0.87835*	0.9097*	0.88235*	0.89536*
	(0.0166)	(0.0149)	(0.01487)	(0.01448)
ξ	-	0.99484*	-	0.96657*
		(0.0239)		(0.02303)
v	5.4243*	4.8882*	5.0283*	5.1121*
	(0.5124)	(0.4346)	(0.4456)	(0.4655)
Diagnostic tests				
Q(10)	0.0188	0.0110	0.0226	0.0492
Q(15)	0.0221	0.0185	0.0276	0.0714
Q(20)	0.0298	0.0257	0.0303	0.1006
$Q^2(10)$	0.4130	0.2255	0.5430	0.7172
$Q^2(15)$	0.5339	0.4351	0.7190	0.8379
$Q^2(20)$	0.6751	0.4643	0.8297	0.8867
LM ARCH	0.4124	0.3684	0.5936	0.7861
Information criterion				
AIC	-6.998	-6.944	-6.929	-6.932
BIC	-6.987	-6.929	-6.912	-6.917
SIC	-6.998	-6.944	-6.929	-6.932
HQIC	-6.994	-6.939	-6.923	-6.926

Coefficient values are significant at 1%*, 5%** and 10%***. Standard errors are provided in parentheses. Results provided in diagnostic tests are *p*-values of the respective test statistics. LM ARCH is the Lagrange Multiplier test for ARCH effects

(greater than 1%). According to Ruppert (2011), large sample sizes often have small or statistically significant *p*-values at small autocorrelations. The Q-statistics for squared residuals are obviously insignificant (higher *p*-values), indicating that the GARCH(1,1) models successfully capture the volatility component in the series. The Lagrange Multiplier (LM) test for ARCH effects for each series is insignificant, and hence, rejecting the null hypothesis of no presence of ARCH effects in the series. The final indicator of the best-fit model is the information criterion, where the best model has the smallest information criterion.

The standardized residual series from the marginal distribution models are then transformed into pseudo observations. The estimated Kendall's tau, which is the copula-based dependence measure, are 0.7060677 for KLCI-Hijrah Shariah and 0.7891004 for EMAS and its Shariah counterpart. Both values are significant at 1% level. Using the Kendall's tau, the copula parameter of each family is then estimated. Table 3 presents the estimated copula parameters and the statistic and *p*-values of the goodness-of-fit test. The estimated copula parameters are higher for the EMAS-EMAS Shariah pair compared to the KLCI-Hijrah Shariah pair, which is also reflective of the stronger association of the EMAS-EMAS Shariah pair measured by the non-parametric correlation in Table 1.

The elliptical copulas (normal and student's *t* copula) for the KLCI-Hijrah Shariah pair have insignificant GOF statistics ($p > 0.01$), suggesting that the elliptical copulas provide good fits in representing the dependence structure of the KLCI and Hijrah Shariah. The student's *t* copula is chosen as the best copula model for the KLCI-Hijrah Shariah because of the higher *p*-value (or higher insignificance) of the GOF statistic. This result suggests that the joint distribution of the KLCI and Hijrah Shariah has a symmetric dependence where the extreme dependence exists at both

TABLE 3. Copula parameter estimates and the results of goodness-of-fit (GOF) test

	KLCI & Hijrah Shariah		EMAS & EMAS Shariah	
Copula	Parameter	GOF	Parameter	GOF
Normal	0.8953	0.0106 [0.5150]	0.9456	0.0120 [0.1653]
Student's *t*	0.8951 (7.9971)	0.0109 [0.5280]	0.9457 (4.7208)	0.0081 [0.6079]
Clayton	4.8043	0.7270 [0.0005]	7.4832	0.5838 [0.0005]
Gumbel	3.4021	0.1412 [0.0005]	4.7416	0.0927 [0.0005]
Frank	11.6945	0.2157 [0.0005]	17.1469	0.2066 [0.0005]
Galambos	2.6921	0.1427 [0.0005]	4.0322	0.0935 [0.0005]
Husler-Reiss	3.4190	0.1488 [0.0005]	4.9411	0.0983 [0.0005]

For Student's *t* copula, the value in parenthesis is the estimated parameter value for ν. The numbers provided for GOF test are the statistics and *p*-values (in square brackets)

tails of the joint distribution. The results also indicate that the returns of KLCI and Hijrah Shariah comove together during bull and bear market conditions.

For the EMAS and EMAS Shariah pair, the Archimedean and extreme-value copulas have the smallest *p*-values of the GOF test, indicating that these copulas are unsuitable to model the dependence of EMAS and its Shariah counterpart. In other words, the dependence structure of the paired indices is not asymmetric and the tail dependence is not one-sided. The student's *t* copula is found to be a good fit for the paired indices since the *p*-value of the GOF test is the largest compared to others. Similar to the KLCI-Hijrah Shariah pair, the EMAS-EMAS Shariah pair also has a symmetric dependence and has extreme dependencies at both upper and lower tails. Therefore, we can conclude that the Islamic and conventional indices of both pairs behave in a similar manner, which are consistent with the results of Albaity and Ahmad (2008) who found that the returns of KLSI and KLCI move in the same direction in the long-run period. There are two possible reasons for our findings. Firstly, the Islamic and conventional markets in Malaysia are driven by common risk factors in most cases (in both crisis and non-crisis periods), and secondly, large portions of companies under the Hijrah Shariah and EMAS Shariah are listed under the KLCI and EMAS, respectively.

In terms of tail dependence, the estimated index for KLCI-Hijrah Shariah and EMAS-EMAS Shariah are 0.4982652 and 0.703856, respectively. The significance of the tail dependencies indicates that both Islamic and conventional stock market returns reacts similarly during financial crises (bear market) and market blooming (bull market) conditions. Both the overall and tail dependences have strong and positive values where the EMAS-EMAS Shariah pair has stronger overall and extreme dependencies compared to the KLCI-Hijrah Shariah pair. One plausible reason for these findings is that the proportion of Shariah-compliant companies in the EMAS index is higher than the proportion of Shariah-compliant companies in the KLCI index. Figure 2, which shows the empirical plot of pseudo observations for each paired indices, further confirm the strong extreme dependences at both the upper and lower tails of the KLCI-Hijrah Shariah and EMAS-EMAS Shariah pairs.

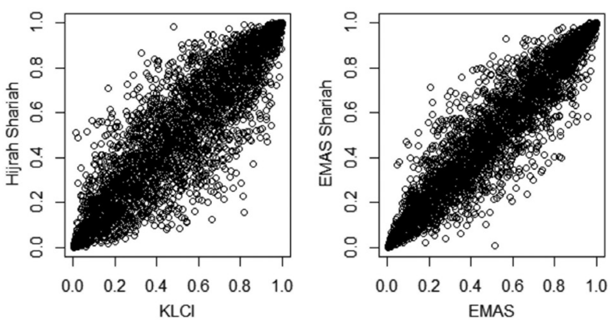

FIGURE 2. Scatterplots of pseudo observations

The portfolio risk of the two portfolios, namely the KLCI-Hijrah Shariah and the EMAS-EMAS Shariah, are then evaluated using the portfolio value-at-risk (VaR) measure. The estimated portfolio VaR from the copula-GARCH model, which from here onwards will be referred as the copula-VaR estimate, is also compared to the portfolio VaR obtained from the variance-covariance method at several levels of investor's risk aversion, α (1%, 5% and 10%). The copula-VaR approach is a two-stage procedure which involves: Generating the portfolio returns using the best-fit ARMA(p,q)-GARCH(1,1)-t-copula models; and Evaluating the portfolio VaR by taking the α-th quantile of the portfolio return distribution.

The accuracy of the portfolio Var estimates are then ensured by performing backtesting where the one-day-ahead portfolio VaR is evaluated for every 250 days and this procedure continues for the next 2984 days (in-sample risk forecast). For each of the 2985 days, the predicted VaR estimates are compared with the observed (actual) portfolio returns, where the number of exceedances is recorded for calculating the violation ratio (VR).

Table 4 shows the expected number of exceedances (exp), the observed number of exceedances (obs), the violation ratio (VR), and the *p*-values of Kupiec's proportion of failure tests (UC). It should be noted that the mixed-weightage portfolio considers a heavier weight contributed by the Shariah index where the proportions are 30% conventional and 70% Shariah indices. A VaR model provides a good risk estimate if the number of exceedances from the VaR model is closer to the expected number of exceedance, or if the VR is within the range of [0.8,1.2], or if the null hypothesis of the Kupiec's test is not rejected (large *p*-value or large UC).

The results showed that the observed number of VaR exceedances from the normal VaR model underestimate the expected number of exceedances at most levels (5% and 10%), while the copula VaR model overestimate the expected number of exceedances at all levels (1%, 5% and 10%). Based on the closeness of the number of VaR exceedances from the expected number of exceedances, the copula-VaR provides better risk measures at 1% and 10% levels while the normal-VaR has better risk measures at 5% level for both KLCI-Hijrah Shariah and EMAS-EMAS Shariah pairs under both equally- and mixed-weighted portfolios.

The VRs, which should be in the range of 0.8 to 1.2, indicate that the copula-VaR model has acceptable risk measures at 5% level (EMAS-EMAS Shariah) and at 10% level (KLCI-Hijrah Shariah and EMAS-EMAS Shariah) under both equally- and mixed-weighted portfolios. On the other hand, the normal VaR model provides acceptable risk measures at 5% level (KLCI-Hijrah Shariah and EMAS-EMAS Shariah) only under the mixed-weighted portfolio.

Based on the Kupiec's test, the copula-VaR model provides insignificant *p*-values (larger than 0.01) at 10% level (KLCI-Hijrah Shariah and EMAS-EMAS Shariah) under the equally-weighted portfolio, and at all levels (EMAS-EMAS Shariah) under the mixed-weighted portfolio. The normal-VaR provides insignificant *p*-values at 10% level (KLCI-Hijrah Shariah and EMAS-EMAS Shariah) under the equally-weighted portfolio and at 5% level (KLCI-Hijrah Shariah and EMAS-EMAS Shariah) under the mixed-weighted portfolio.

The overall results showed that the copula-VaR model consistently provides better risk estimates than the normal VaR at 10% level. However, the better models at 1% and 5% levels are indecisive based on the strict statistical testing of the VR and the Kupiec's test. The results also indicate that investing a larger proportion in Shariah index is not that different than investing in an equally-proportioned portfolio since both portfolios produce similar VaR measures in terms of the observed number of VaR exceedances versus the expected number of exceedances.

CONCLUSION

This study has modeled the dependence between Islamic and conventional indices in Malaysia in years 2000-2012 using copula approach with volatility models as marginal distributions, and evaluates the portfolio risks of both

TABLE 4. VaR and backtesting results

		Normal VaR			Copula-VaR		
		Obs.	VR	UC	Obs.	VR	UC
5:5 weighted portfolio							
KLCI-Hijrah Shariah	VaR 1%	61	2.1034	0.0000	48	1.3020	0.0003
	VaR 5%	135	0.7483	0.0000	194	1.2517	0.0000
	VaR 10%	223	1.4483	0.0352	373	0.9060	0.2242
EMAS-EMAS Shariah	VaR 1%	63	2.1724	0.0000	50	1.2819	0.0008
	VaR 5%	129	0.7114	0.0000	191	1.1812	0.0015
	VaR 10%	212	1.3448	0.1080	352	0.8658	0.0820
3:7 weighted portfolio							
KLCI-Hijrah Shariah	VaR 1%	61	2.1034	0.0000	45	1.5517	0.0095
	VaR 5%	136	0.9128	0.2589	178	1.2282	0.0061
	VaR 10%	223	0.7483	0.0000	335	1.1846	0.0012
EMAS-EMAS Shariah	VaR 1%	63	2.1724	0.0000	42	1.4483	0.0232
	VaR 5%	132	0.8859	0.1398	178	1.1946	0.0190
	VaR 10%	216	0.7248	0.0000	335	1.1242	0.0286

indices using the copula-VaR model and different mixed-weighted portfolios. The findings from this study showed that the ARMA-GARCH models with student-*t* and skewed student-*t* distributions are sufficient to model the marginals of return series of the conventional (or non-Shariah) and Shariah indices, respectively. Student's *t* copula was found as the best copula model for both KLCI-Hijrah Shariah and EMAS-EMAS Shariah pairs, suggesting that the joint distribution of both pairs have symmetric dependences where the extreme dependences exist at both upper and lower tails of the joint distributions, or the returns of KLCI-Hijrah Shariah and EMAS-EMAS Shariah comove together during both bull and bear market conditions. In addition, the significance of the estimated tail dependence index for the KLCI-Hijrah Shariah and EMAS-EMAS Shariah implies that both Islamic and conventional stock market returns reacts similarly during financial crises (bear market) and market blooming (bull market) conditions. Besides that, the overall results of the portfolio risk assessments with backtesting indicate that the copula-VaR model consistently provides better risk estimates than the normal VaR at 10% level, but the better models at 1% and 5% levels are indecisive when strict statistical tests of the VR and Kupiec's test are utilized. However, further investigations using other analyses are required before reaching to this conclusion. For future studies, the scope of this study can be extended by using advanced GARCH models and dynamic copulas (time-variant); adding new insights in terms of changes in the tail dependences with respect to time. In addition, weekly data can used for robustness testing.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support from the Ministry of Higher Education Malaysia, under the FRGS and GUP grants (FP055-2014A, FRGS/1/2014/SG04/UM/01/1, FRGS/1/2015/SG04/UKM/02/2 and GUP-2015-002) and Multimedia University (MMUI/170067).

REFERENCES

Ab Razak, R. & Ismail, N. 2016. Portfolio risks of bivariate financial returns using copula-VaR approach: A case study on Malaysia and U.S. stock markets. *Global Journal of Pure and Applied Mathematics* 12(3): 1947-1964.

Ab Razak, R. & Ismail, N. 2015. The dependence of Islamic and conventional stocks: A copula approach. *AIP Conference Proceeding* 1678 060012: 1-5.

Albaity, M. & Ahmad, R. 2008. Performance of Syariah and composite indices: Evidence from Bursa Malaysia. *Asian Academic of Management Journal of Accounting and Finance* 4(1): 23-43.

Aloui, R., Ben Aïssa, M.S., Hammoudeh, S. & Nguyen, D.K. 2014. Dependence and extreme dependence of crude oil and natural gas prices with applications to risk management. *Energy Economics* 42: 332-342.

Genest, C., Rémillard, B. & Beaudoin, D. 2009. Goodness-of-fit tests for copulas: A review and a power study. *Insurance: Mathematics and Economics* 44(2): 199-213.

Ghalanos, A. 2015. Introduction to the rugarch package (Version 1.3-1). https://cran.r-project.org/web/packages/rugarch/vignettes/Introduction_to_the_rugarch_package.pdf. Accessed on 27 January 2016.

Hammoudeh, S., Mensi, W., Reboredo, J.C. & Nguyen, D.K. 2014. Dynamic dependence of the global Islamic equity index with global conventional equity market indices and risk factors. *Pasific-Basin Finance Journal* 30: 189-206.

Heinen, A. & Valdesogo, A. 2012. Copula-based volatility models. In. *Handbook of Volatility Models and Their Applications*, edited by Bauwens, L., Hafner, C. & Laurent, S. New Jersey: John Wiley & Sons. pp. 293-316.

Jäschke, S. 2014. Estimation of risk measures in energy portfolios using modern copula techniques. *Computational Statistics and Data Analysis* 76: 359-376.

Melo Mendes, B.V.D. & Aíube, C. 2011. Copula based models for serial dependence. *International Journal of Managerial Finance* 7(1): 68-82.

Nelson, R.B. 2006. *An Introduction to Copulas*. New York: Springer.

Ning, C. 2010. Dependence structure between the equity market and the foreign exchange market - A copula approach. *Journal of International Money and Finance* 29: 743-759.

Posedel, P. 2005. Properties and estimation of GARCH (1,1) model. *Metodološki Zvezki* 2(2): 243-257.

Riddles, N. 2001. A portfolio manager's view of downside risk. In. *Managing Downside Risk in Financial Markets*, edited by Sortino, F. & Satchell, S. Oxford: Butterworth-Heinemann Finance. pp. 74-92.

Righi, M.B. & Ceretta, P.S. 2013. Estimating non-linear serial and cross-interdependence between financial assets. *Journal of Banking and Finance* 37: 837-846.

Righi, M.B. & Ceretta, P.S. 2011. Modeling risk dependence with copulas in financial market: A Latin case. *European Journal of Scientific Research* 59(3): 295-306.

Ruppert, D. 2011. *Statistics and Data Analysis for Financial Engineering*. New York: Springer.

Shamiri, A., Hamzah, N.A. & Pirmoradian, A. 2011. Tail dependence estimate in financial market risk management: Clayton-Gumbel copula approach. *Sains Malaysiana* 40(8): 927-935.

Stoyanov, S.V., Rachev, S.T., Racheva-Yotova, B. & Fabozzi, F.J. 2011. Fat-tailed model for risk estimation. *The Journal of Portfolio Management* 37(2): 107-117.

Ruzanna Ab Razak*
Faculty of Management
Multimedia University
63100 Cyberjaya
Selangor Darul Ehsan
Malaysia

Noriszura Ismail
Pusat Pemodelan dan Sains Data
Fakulti Sains dan Teknologi
Universiti Kebangsaan Malaysia
43600 UKM Bangi, Selangor Darul Ehsan
Malaysia

*Corresponding author; email: ruzanna415@gmail.com

Received: 21 July 2017
Accepted: 3 May 2019