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Unsteady Transport Phenomena of Hybrid Al_2O_3-Cu/H_2O Nanofluid Past a Shrinking Slender Cylinder

(Fenomenon Angkutan Tak Mantap Nanobendalir Hibrid Al_2O_3-Cu/H_2O terhadap Silinder Langsing Mengecut)

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ABSTRACT

Theoretical investigations of unsteady boundary layer flow gain interest due to its relatability to practical settings. Thus, this study proposes a unique mathematical model of the unsteady flow and heat transfer in hybrid nanofluid past a permeable shrinking slender cylinder. The suitable form of similarity transformations is adapted to simplify the complex partial differential equations into a solvable form of ordinary differential equations. A built-in bvp4c function in MATLAB software is exercised to elucidate the numerical analysis for certain concerning parameters, including the unsteadiness and curvature parameters. The bvp4c procedure is excellent in providing more than one solution once sufficient predictions are visible. The present analysis further observed dual solutions that exist in the system of equations. Notable findings showed that by increasing the nanoparticles volume fraction, the skin friction coefficient increases in accordance with the heat transfer rate. In contrast, the decline of the unsteadiness parameter demonstrates a downward trend toward the heat transfer performance.

Keywords: Dual solutions; hybrid nanofluid; shrinking cylinder; unsteady flow

ABSTRAK

Kajian teori aliran lapisan sempadan tak mantap telah menarik minat ramai kerana kaitannya dengan tetapan praktikal. Oleh itu, kajian ini mencadangkan suatu model matematik yang unik untuk mengkaji aliran tak mantap dan pemindahan haba dalam nanobendalir hibrid melewati silinder langsing telap mengecut. Bentuk penjelmaan keserupaan yang sesuai digunakan untuk menurunkan persamaan pembezaan separa yang kompleks kepada bentuk persamaan pembezaan biasa yang boleh diselesaikan. Fungsi bvp4c terbina dalam perisian MATLAB digunakan untuk menjelaskan analisis berangka bagi parameter yang bersangkutan termasuk parameter ketakmantapan dan kelengkungan. Prosedur bvp4c adalah sangat sesuai dan cekap dalam memperoleh lebih daripada satu penyelesaian dengan ramalan yang mencukupi. Analisis kajian ini mendapati yang penyelesaian dual wujud dalam sistem persamaan. Hasil keputusan kajian menunjukkan bahawa dengan meningkatkan pecahan isi padu nanozarah, pekali geseran kulit meningkat sesuai dengan kadar pemindahan haba. Sebaliknya, penurunan parameter ketakmantapan menunjukkan trend penurunan prestasi pemindahan haba.

Kata kunci: Aliran tak mantap; nanobendalir hibrid; penyelesaian dual; silinder mengecut

INTRODUCTION

Human society resides in advanced technology where heat transfer is essential in various sectors, including power generation, and electronic devices. This continuous development includes progressive system operating and performance, which pursues a new cooling technology revolution. In this case, such instruments' thermal management is remarkable, insisting that the researchers create an efficient medium for heat transfer. Today, a revolutionary type of nanofluid, i.e. hybrid nanofluid, is adopted to embrace the required enhancement in the traditional fluid's heat-conducting. This innovative formula of nanofluid is aimed to provide considerably evolved thermal conductivity. Shortly after the impressive observations made by Choi and Eastman (1995) that gave rise to the genius idea of nanofluid, this modern form of a working fluid, i.e. hybrid nanofluid, has now fascinated many researchers. Das (2017) reviewed the impact of thermal conductivity between conventional nanofluids and hybrid nanofluids. The study reported that the hybrid nanofluid possesses enhanced thermal conductivity characteristics compared to conventional nanofluids.

Hamzah et al. (2017) concluded that the hybrid nanofluid's efficiency is usually defined by its dispersion's stability, volume concentration and mixing ratios. The latest study on the application of hybrid nanofluids has been examined by Jamil and Ali (2020), Shah and Ali (2019) and Yang et al. (2020). Additionally, numerous researchers have endeavored to develop heat transfer characteristics and boundary layer beyond hybrid nanofluid with various physical conditions. Nadeem et al. (2020) evaluated the performance of hybrid nanofluid toward a curved surface while the influence of magnetic field and thermal radiation of Al_2O_3-Cu/H_2O on a permeable moving surface is addressed by Zainal et al. (2020a). Khashi'ie et al. (2020a) studied the Marangoni convection flow of Al_2O_3-Cu/H_2O over a permeable stretching/shrinking sheet. Waini et al. (2020a) explored the flow of a stretching/shrinking cylinder intruding on a stagnation point while Khashi'ie et al. (2020b) examined the heat transfer in Al_2O_3-Cu/H_2O toward a shrinking cylinder by taking into account the enforcement of prescribed surface heat flux. Some available reviews focused on the flow of a deformable cylinder can be assessed through Hosseinzadeh et al. (2020), Khashi'ie et al. (2019), Maskeen et al. (2019), and Salleh and Nazar (2010).

In various manufacturing systems, practitioners prefer a continuous steady flow, so they have greater control over the operations. However, the unintended unsteady effect may take place around a system even in an ideal flow environment. The previous investigation reported that the unsteady flow activity reflects an odd pattern instead of the steady flow due to the different time-based conditions, which distorted boundary layer distinction and fluid motion structure (Smith 1986; White 1991). Nevertheless, with a comprehensive understanding of the unsteady flow application in manufacturing processes, the new design approaches that facilitate system performance, reliability, and cost reduction of many fluid dynamic devices are feasible (McCroskey 1977). To the best of our knowledge, Benson et al. (1964) initiated the numerical study of unsteady flow problems while Sears and Telionis (1975) analysed the boundary layer separation in unsteady flow. Fang et al. (2011) investigated the unsteady viscous flow past an expanding stretching cylinder while Zaimi et al. (2017) performed a numerical analysis of unsteady flow over a permeable shrinking cylinder in nanofluid using Buongiorno's model. Since then, a significant series of researches have been conducted concerning unsteady flow and heat transfer in several aspects including the stretching/shrinking sheet, for instance, Jahan et al. (2018), Naganthran et al. (2020) and Zainal et al. (2020b, 2020c).

Experts frequently strived to enhance the heat transfer performance toward a stretching/shrinking surface due to the overwhelming applications in the manufacturing industry. For example, drawing plastic films, glass blowing, hot rolling, and plastic sheets extrusion (Altan et al. 1983; Fisher 1976; Kamal et al. 2019). According to the research done by Karwe and Jaluria (1991), the final product consistency with optimal qualities relies on heat-conductivity between the fluid flow and stretching/shrinking sheet. Wang (1988) was the pioneer who initiates the fluid flow analysis past a shrinking cylinder. His research was further explored by Abbas et al. (2015), Butt and Ali (2014) and Vajravelu et al. (2012).

The study of fluid flow toward a cylinder is crucial and essential to the industries, especially inflow prediction, heat transfer and pollutant diffusion about invasive bodies, particularly piping intrusions of a magnet, casting procedures and salt domes (Ganesan & Loganathan 2003). Moreover, in the geological formation's studies, including the mining and heat restoration of oil, underwater radioactive waste disposal facilities, and geothermal reservoirs, the flow problem past a cylinder has significant applications. According to Vajravelu and Mukhopadhyay (2016), in several manufacturing applications, including wire and fibre drawing, the effect of transverse curvature is crucial where precise prediction is required and a thick boundary layer may occur on slender or near slender bodies. The curvature parameter occurs in the governing equation when the cylinder is thin/slender. Bachok and Ishak (2010) investigated the flow and heat transfer over a horizontal cylinder with the influence of the prescribed surface heat flux and transverse curvature parameter. They found that the addition of the curvature parameter successfully increased the heat transfer performance. Later, Najib et al. (2014) analyzed the impact of a chemical reaction and curvature parameter on the stagnation point flow and mass transfer over a stretching/shrinking cylinder. Their study indicated that as the curvature parameter increased, the surface shear stress and the mass transfer rate improved. The perseverance of flow analysis over the stretching/shrinking cylinder has been configured by countless researchers recently, including Almaneea (2020), Awaludin et al. (2020), Khashi'ie et al. (2020c), Roşca et al. (2020), and Tabassum et al. (2020).

A detailed evaluation of the previously mentioned literature remains to a minimal extent. As far as we are aware, an investigation of the unsteady transport phenomena of hybrid nanofluid towards shrinking cylinder still lacks in the scientific literature. Therefore, driven by the stated key questions, the current effort intends to construct a mathematical hybrid nanofluid model by executing a numerical analysis using the Tiwari and Das (2007) model unsteady flow and heat transfer past a permeable shrinking slender cylinder. The thermophysical properties of hybrid nanofluids were applied based on Ghalambaz et al. (2020) and Takabi and Salehi (2014) work. Besides, the current study employed the bvp4c function in the MATLAB software systems to solve the formulated problem. This approach method has successfully established more than one solution, along with the permeable shrinking cylinder. Also, the comparison outcomes for restricting cases are conducted and reveals excellent agreement with the already existing data.

MATHEMATICAL MODELLING

An unsteady flow of a hybrid nanofluid past a permeable shrinking slender cylinder is analysed in this study. Figure 1 demonstrates the schematic problem flow, where (x,r) is the coordinate system and the working fluid is supposed to flow in the $x-$ axis while the $r-$ coordinate is normal to it. It is assumed that the deformable (stretching/shrinking) circular cylinder has a constant radius a and consist of linear velocity $U_w(x)$ with uniform characteristic velocity u_0 where $U_w(x)=u_o x/L(1-\beta t)$. In addition, the constant mass flux is indicated by $v_w(r)$, thus $v_w(r)<0$ and $v_w(r)>0$ signifies the suction and injection condition, respectively. From these assumptions, it is possible to describe the governing boundary layer equations as (Khashi'ie et al. 2020b; Tiwari & Das 2007; Waini et al. 2020b):

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\frac{\mu_{hnf}}{\rho_{hnf}}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right), \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{k_{hnf}}{(\rho C_p)_{hnf}}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right), \tag{3}$$

subject to the boundary conditions:

$$\begin{aligned} &u=\lambda U_w(x),\ v=v_w(r),\ T=T_w(x),\ \text{at } r=a,\\ &u\to 0,\ T\to T_\infty \ \text{as } r\to\infty. \end{aligned} \tag{4}$$

At this point, u is the velocity components along $x-$ axis and v is the velocity of hybrid nanofluid for $r-$ axis, T is the temperature of Al_2O_3-Cu/H_2O, μ_{hnf} is the dynamic viscosity of Al_2O_3-Cu/H_2O, ρ_{hnf} the density of Al_2O_3-Cu/H_2O, while k_{hnf} and $(\rho C_p)_{hnf}$ is the thermal conductivity and heat capacity of Al_2O_3-Cu/H_2O, respectively. It is assumed that $T_w(x)=T_\infty+\left(T_0(x/L)^2/(1-\beta t)^2\right)$, where T_0 is the characteristic temperature and L is the characteristic length. The constant stretching/shrinking parameter is symbolized by λ with $\lambda>0$ and $\lambda<0$ stand for stretching and shrinking cylinder, respectively, while $\lambda=0$ indicated a rigid cylinder. Further, Table 1 offers the nanoparticles thermophysical properties of H_2O (water), Al_2O_3 (alumina), and Cu (copper) as demonstrated by Oztop and Abu-Nada (2008). Meanwhile, the correlation coefficient of Al_2O_3-Cu/H_2O described by Ghalambaz et al. (2020) and Takabi and Salehi (2014) is presented in Table 2.

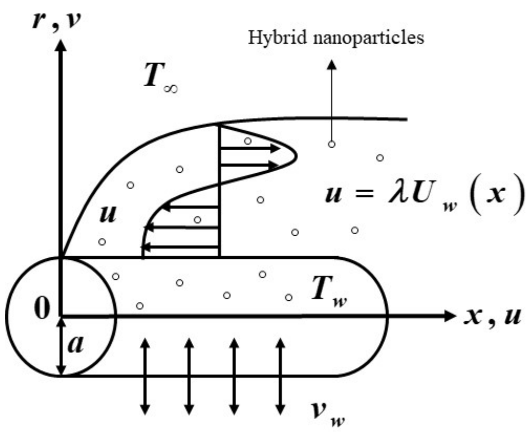

FIGURE 1. The schematic problem flow of shrinking cylinder

TABLE 1. The H_2O, Al_2O_3 and Cu thermophysical properties

Physical properties	H_2O	Al_2O_3	Cu
$\rho(kg/m^3)$	997.1	3970	8933
$C_p\,(J/kgK)$	4179	765	385
$k\,(W/mK)$	0.613	40	400

TABLE 2. The alumina-copper/water (Al_2O_3-Cu/H_2O) correlation coefficient

Properties	Al_2O_3-Cu/H_2O
Density	$\rho_{hnf}=\left(1-\phi_{hnf}\right)\rho_f+\phi_1\rho_{s1}+\phi_2\rho_{s2}$
Thermal capacity	$\left(\rho C_p\right)_{hnf}=\left(1-\phi_{hnf}\right)\left(\rho C_p\right)_f+\phi_1\left(\rho C_p\right)_{s1}+\phi_2\left(\rho C_p\right)_{s2}$
Dynamic viscosity	$\mu_{hnf}=\dfrac{1}{\left(1-\phi_{hnf}\right)^{2.5}}$
Thermal conductivity	$\dfrac{k_{hnf}}{k_f}=\left[\dfrac{\left(\dfrac{\phi_1k_{s1}+\phi_2k_{s2}}{\phi_{hnf}}\right)+2k_f+2\left(\phi_1k_{s1}+\phi_2k_{s2}\right)-2\phi_{hnf}k_f}{\left(\dfrac{\phi_1k_{s1}+\phi_2k_{s2}}{\phi_{hnf}}\right)+2k_f-\left(\phi_1k_{s1}+\phi_2k_{s2}\right)+\phi_{hnf}k_f}\right]$

The following similarity variables are employed:

$$u=\frac{u_0x}{L(1-\beta t)}f'(\eta),\quad v=-\frac{a}{r}\sqrt{\frac{u_0\nu_f}{L(1-\beta t)}}f(\eta),$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\eta=\sqrt{\frac{u_0}{\nu_fL(1-\beta t)}}\left(\frac{r^2-a^2}{2a}\right),\qquad(5)$$
$$\text{hence }\ v_w=-\frac{a}{r}\sqrt{\frac{u_0\nu_f}{L(1-\beta t)}}S,$$

where S is the mass flux velocity with $S>0$ for suction and $S<0$ for injection or withdrawal of the fluid. Now by substituting (5) into (2)−(4), the following equations are formulated:

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}\left[(1+2\gamma\eta)f'''+2\gamma f''\right]+\left(f-\frac{\varepsilon}{2}\eta\right)f''-(\varepsilon+f')f'=0,\ (6)$$
$$\frac{1}{\Pr}\frac{k_{hnf}/k_f}{(\rho C_p)_{hnf}/(\rho C_p)_f}\left[(1+2\gamma\eta)\theta''+2\gamma\theta'\right]+\left(f-\frac{\varepsilon}{2}\eta\right)\theta'-2(\varepsilon+f')\theta=0,(7)$$

subject to the boundary conditions:

$$f(0)=S,\ f'(0)=\lambda,\ \theta(0)=1,$$
$$f'(\eta)\to0,\ \theta(\eta)\to0,\ \text{as}\ \eta\to\infty\qquad(8)$$

where primes denote differentiation with respect to η, $\gamma=\sqrt{\nu_fL(1-\beta t)/u_0a^2}$ is the curvature parameter, $\Pr=\mu_fC_p/k_f$ is the Prandtl number, and $\varepsilon=\beta L/u_0$ represents the unsteadiness parameter, with $\varepsilon>0$ denotes the accelerating flow while $\varepsilon<0$ corresponds to decelerating flow. In this study, we assume flow over decelerating stretching/shrinking sheet with $\varepsilon<0$ The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which is defined as:

$$C_f=\tau_w/\rho_fU_w^{\,2},\ Nu_x=\frac{xq_w}{k_f\left(T_w-T_\infty\right)},\qquad(9)$$

where τ_w is the skin friction or shear stress and q_w is the heat flux along the cylinder where:

$$\tau_w=\mu_{hnf}\left(\frac{\partial u}{\partial r}\right)_{r=a},\ q_w=-k_{hnf}\left(\frac{\partial T}{\partial r}\right)_{r=a}.\qquad(10)$$

Using (5), (9) and (10), we get:

$$\mathrm{Re}_x^{1/2}C_f=\frac{\mu_{hnf}}{\mu_f}f''(0),\quad \mathrm{Re}_x^{-1/2}Nu_x=-\frac{k_{hnf}}{k_f}\theta'(0),\ (11)$$

where $\mathrm{Re}_x = u_0 x^2 / v_f L(1-\beta t)$ is the local Reynolds number.

Results and Discussion

The resulting nonlinear ordinary differential equations presented in (6)-(7) in common with the boundary conditions (8) were elucidated through the bvp4c feature in the MATLAB systems software (Shampine et al. 2003). The expressed boundary value problem is demoted to an ordinary differential equations system of first-order, initially. The bvp4c method is a noteworthy solver used by numerous researchers to justify the boundary value problem. An initial prediction of the principal mesh point and variations step size is necessary advantageous to the required solutions assurance. In an attempt to discover more than one solution, the appropriate estimation of boundary layer thickness is important. The users also need to provide several trials in supplying an excellent preliminary guess before a necessary result is obtained. The reliability of the findings is measured with Devi and Devi (2017), Khashi'ie et al. (2020d) and Waini et al. (2019), as accessible in Table 3. The authors specifically found that the current results conform to the previous work remarkably. Therefore, we claim that the projected computational framework can be fully implemented to evaluate the heat transfer and fluid flow behaviors with considerable certainty.

TABLE 3. Approximation values of $\mathrm{Re}_x^{1/2} C_f$ by certain values of ϕ_2 when $\varepsilon = \gamma = S = 0, \phi_1 = 0.1, \lambda = 1$ and $\mathrm{Pr} = 6.135$.

ϕ_2	Present result	Khashi'ie et al. (2020d)	Waini et al. (2019)	Devi and Devi (2017)
0.005	−1.328754	−1.327098	−1.327098	−1.327310
0.02	−1.416377	−1.409490	−1.409490	−1.409683
0.04	−1.535214	−1.520721	−1.520721	−1.520894
0.06	−1.657219	−1.634119	−1.634119	−1.634279

The preparation of compatible mono/hybrid nanofluids is the main element for measuring nanofluids' flow behavior and the efficiency of heat transfer. The hybrid Al_2O_3-Cu/H_2O nanofluid is selected in this study because of the outstanding work of Suresh et al. (2011a) in developing an exploration practice to scrutinize the Al_2O_3-Cu/H_2O thermophysical properties. In another study, Suresh et al. (2011b) performed the synthesis of Al_2O_3-Cu/H_2O nanocomposite powder and its characteristics for various volume concentrations. In their noteworthy study, nanofluid stability is observed to decrease as volume concentration increases. Further, according to Arifin et al. (2011), the development of stable nanofluids was to optimize the thermal properties of operating fluids with the minimal use of nanoparticles. Devi and Devi (2016a, 2016b) reported that the Al_2O_3-Cu/H_2O is set up by dissolving Al_2O_3 and Cu nanoparticle into H_2O with various sums of volume fractions. A different range of ϕ_2 values is implemented $(0.00 \le \phi \le 0.02)$, while ϕ_1 is fixed at 0.01. Also, it deserves declaring that the discrete nanoparticle dispersion (Al_2O_3-Cu) is capable of producing ordinary nanofluids, i.e. Al_2O_3/H_2O and Cu/H_2O. Since this current work is compatible with the water-based nanofluid, the Prandtl number is assigned to 6.2, except for comparisons with the previous data. On another point, different values of the limiting parameters are defined to the preceding scope; $-0.1 \le \varepsilon \le -0.5$ and $0.1 \le \gamma \le 0.3$ ensure the consistency of the solutions obtained. Since there is no experimental evidence exist, the choice of parameter values was determined by the values selected from previous researchers to ensure the establishment of the dual solutions (Khashi'ie et al. 2020b, 2020e; Waini et al. 2019). Further, the selection of $\eta_\infty = 15$ certified that all numerical solutions were correctly approaching the asymptotic values. The bvp4c function in the MATLAB programming is utilized in this research work to solve the governing (6)−(7) along with the boundary conditions (8). These proposed approach is a significant technique for tackling the boundary value issues that were thoroughly defined by numerous scholars. In order to accomplish the necessary solution, a suitable preliminary estimation and the boundary layer thickness η_∞ shall be appointed according to the values of the mentioned parameter. Generally, this analysis aims to investigate the input parameter impact including the volume fraction for nanoparticles (ϕ_1, ϕ_2), the unsteadiness parameter (ε), and the curvature parameter (γ) over the physical quantities of interest. The present research also focused to identify the dual solutions which exist in the system of equations. From the generated results of (6) – (8), the non-uniqueness (dual) solutions are perceived to a particular range of λ_c which denotes the meeting point of dual solutions, and this often called a critical point. It is noted that the dual solutions are observed in the shrinking $(\lambda < 0)$ slender cylinder as presented in Figures 2-9, while no dual solutions can be seen as the cylinder stretches $(\lambda > 0)$. The influence of the nanoparticles volume fraction towards the shrinking slender cylinder is presented in Figures 2-5. The skin friction coefficient $(f''(0))$ of a nanofluid $(\phi_1 = 0.01, \phi_2 = 0)$ and hybrid nanofluid $(\phi_1 = 0.01, \phi_2 = 0.005, 0.01)$, along with the heat transfer rate $(-\theta'(0))$ are accessible in Figures 2-3. Figure 2 establishes that the addition of ϕ_2 in the working fluid improves $f''(0)$ whilst the slender cylinder is shrinking. This signifies that the skin friction coefficient of hybrid Al_2O_3-Cu/H_2O nanofluid is higher than Al_2O_3/H_2O nanofluid when 0.5% and 1% of ϕ_2 (Cu) volume concentration is added hence increases the viscosity in the hybrid Al_2O_3-Cu/H_2O nanofluid. This essentially increases the velocity of the fluid past the shrinking slender cylinder, as seen in Figure 4. In the meantime, Figure 3 illustrates a rising trend of $-\theta'(0)$, which denotes the heat transfer rate while the values of ϕ_2 increase in the first solution. Consecutively, the heat transfer efficiency increases in the event that Al_2O_3-H_2O nanofluid shifts into hybrid Al_2O_3-Cu/H_2O nanofluid past a shrinking slender cylinder. This discovery helps to defend the fact that the increase in ϕ_2 hypothetically optimises the heat transfer performance in a hybrid nanofluid compared to the conventional nanofluid. It is also noted that the hybrid nanoparticles have the potential to increase the rate of heat transfer due to synergistic impact, as explained by Sarkar et al. (2015) and this is proven in the hybrid Al_2O_3-Cu/H_2O nanofluid that has been implemented in this study. Furthermore, the profile of velocity $f'(\eta)$, which is presented in Figure 4, shows that the momentum boundary layer thickness was decreased in response to the rise of ϕ_2. This boosts the fluid density and increases the velocity gradient when the slender cylinder is shrunk, eventually. As $f''(0)$ rises, the finding indicates that the frictional drag applied on the unsteady hybrid Al_2O_3-Cu/H_2O nanofluid flow is improved toward the shrinking slender cylinder, which can prolong the separation of the boundary layer flow. On another note, the temperature profiles $\theta(\eta)$ in Figure 5 support the trend seen in Figure 3, which demonstrates the changes in temperature as the ordinary Al_2O_3/H_2O nanofluid turns to hybrid Al_2O_3-Cu/H_2O nanofluid in the shrinking slender cylinder. The decline in hybrid nanofluid temperature decreases the thermal conductivity of the hybrid Al_2O_3-Cu/H_2O nanofluid and gradually increases the convective heat transfer rate. Overall, both profiles, i.e. $f'(\eta)$ and $\theta(\eta)$ asymptotically fulfilled the far-field boundary conditions (8) when $\eta_\infty = 15$ is executed.

Figures 6-7 demonstrate the impact of the unsteadiness parameter ε toward λ as ε varied. The hybrid Al_2O_3-Cu/H_2O nanofluid behavior is depicted in Figure 6 concerning the skin friction coefficient $f''(0)$ when ε varied in the shrinking slender cylinder. Figure 6 captures that as ε reduced, the first solution has decreased in $f''(0)$, and the second solution has exhibited a reverse effect. The decline in ε leads to the expansion of the boundary layer thickness and consequently reduces the velocity gradient of the permeable shrinking slender cylinder, hence $f''(0)$ decreased. The cumulative amount of nanoparticle volume fraction $(\phi_1 = \phi_1 = 0.01)$ in the working fluid, which is 2% could also initiate the reduction of $f''(0)$ due to the increase in the viscosity of hybrid Al_2O_3-Cu/H_2O nanofluid past a shrinking slender cylinder. Moreover, according to the generated results in Figure 7, $-\theta'(0)$ is diminished in the first solution, which is proportional to the rate of heat transfer when ε reduces. From the current and existing evidence, the authors may conclude that the unsteadiness parameter greatly facilitates heat transfer degradation. Even then, if several parameters of regulation are taken into consideration, the authors would also like to suggest that these results can differ.

Figures 8-9 are prepared to show the effect of the curvature parameter γ in relation to λ when $\phi_1 = \phi_2 = 0.01$, which represents the hybrid Al_2O_3–Cu/H_2O nanofluid compound toward a shrinking slender cylinder. Figure 8 highlights that as γ increases, $f''(0)$ decreases in the first solution. Concurrently, Figure 9 presents the heat transfer performance, where $-\theta'(0)$ shows the reduction trend when the values of γ boosts in hybrid Al_2O_3–Cu/H_2O nanofluid while the slender cylinder is shrinking. The findings oppose the claim that the rise in the curvature parameter allegedly increases the efficiency of heat transfer in viscous fluid and nanofluid (Merkin et al. 2017; Rangi & Ahmad 2012). This seems to be due to the fact that the adjustment of the curvature parameter initiates the curvature radius to decrease, thereby decreasing the cylinder area. The cylinder confronts lower tolerance from the fluid particles, escalates the fluid's velocity, and then increases the heat transfer rate. However, we understand that the occurrence of unsteady flow and the adoption of assorted parameters may trigger this behavior. The unsteady parameter may enforce any sum of heat flow in the regulating framework, ultimately aggravating the heat transfer mechanism.

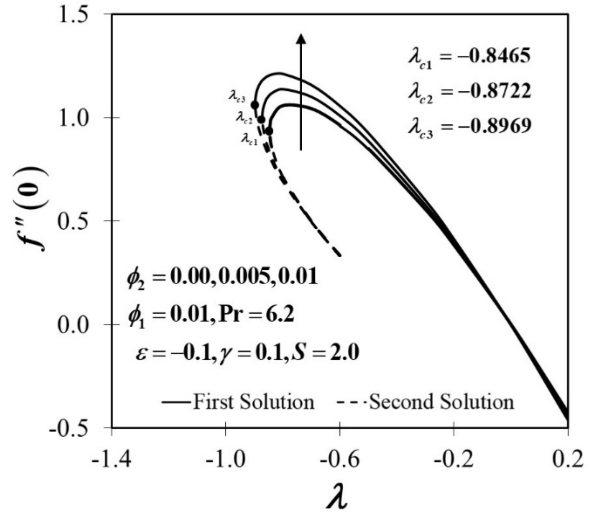

FIGURE 2. $f''(0)$ in relation to λ by various ϕ_2

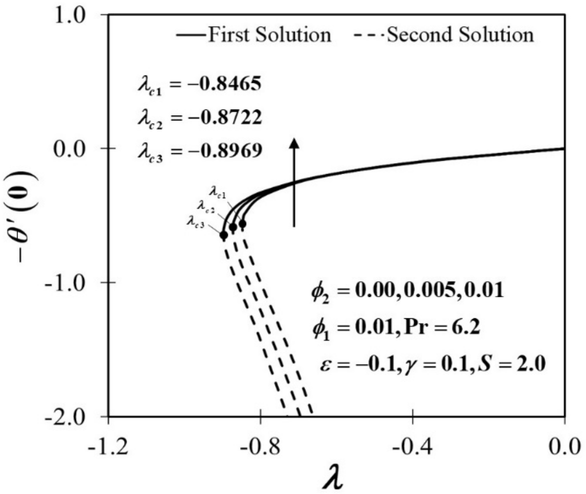

FIGURE 3. $-\theta'(0)$ in relation to λ by various ϕ_2

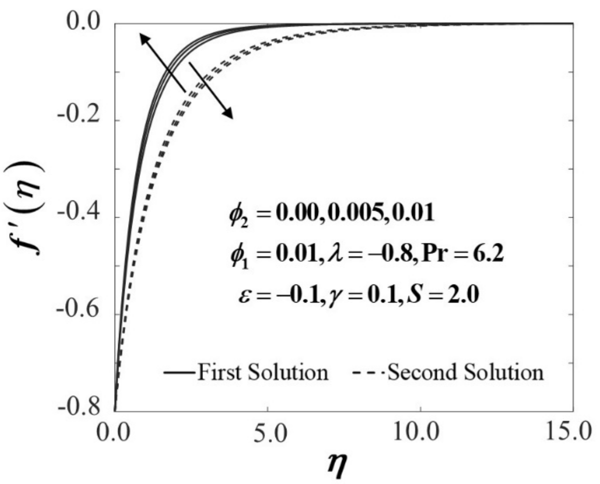

FIGURE 4. $f'(\eta)$ along the shrinking cylinder $(\lambda = -0.8)$

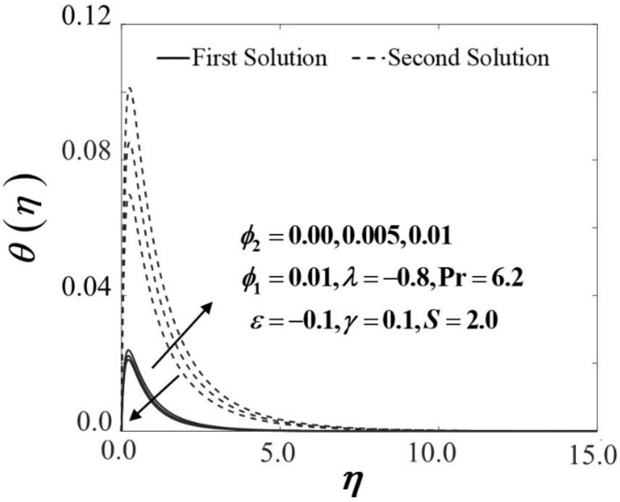

FIGURE 5. $\theta(\eta)$ along the shrinking cylinder $(\lambda = -0.8)$

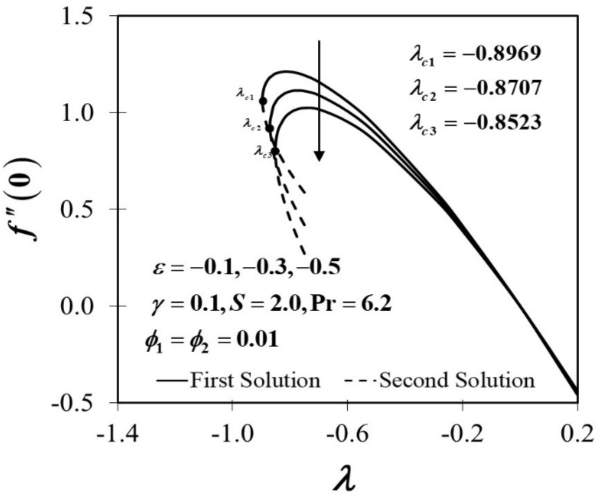

FIGURE 6. $f''(0)$ in relation to λ by various ε

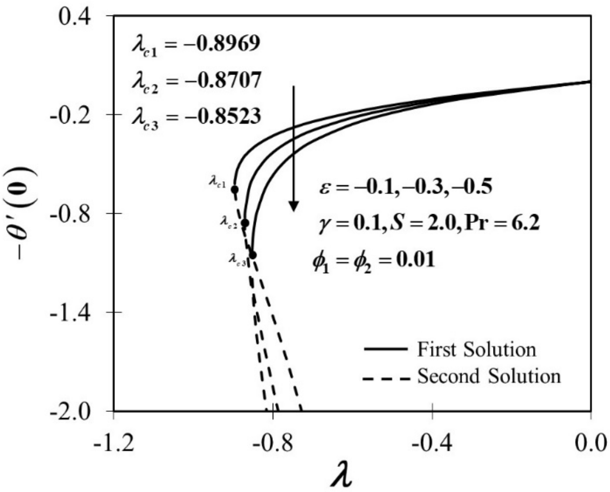

FIGURE 7. $-\theta'(0)$ in relation to λ by various ε

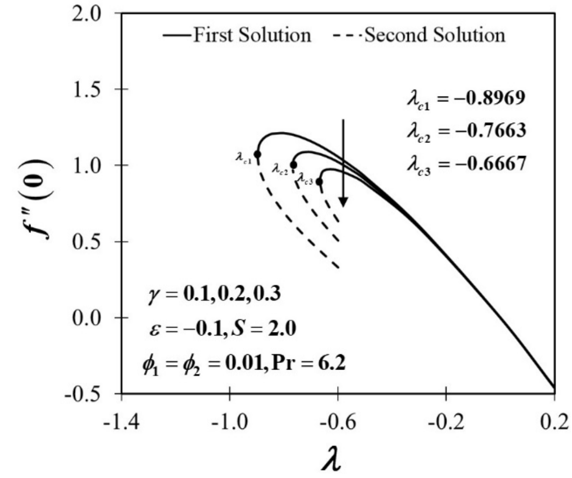

FIGURE 8. $f''(0)$ in relation to λ by various γ

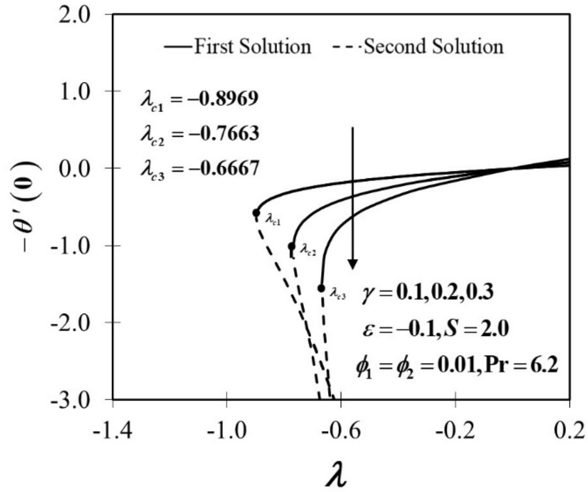

FIGURE 9. $-\theta'(0)$ in relation to λ by various γ

Conclusion

A numerical evaluation on the unsteady flow and heat transfer of hybrid Al_2O_3–Cu/H_2O nanofluid past a shrinking slender cylinder was verified in this current work. The participation of the bvp4c features in the MATLAB programming interface is occupied with executing the numerical analysis. The hybrid Al_2O_3-Cu/H_2O nanofluid is recognised by dissolving the main nanoparticle known as alumina $(\phi_1 = Al_2O_3)$ within the base fluid, which is water (H_2O) and accompanied by copper $(\phi_2 = Cu)$ as the second nanoparticle (ϕ_2) with acceptable proportions of nanoparticles volume fraction. The results of different control parameters were studied, for instance, the nanoparticle volume fraction parameter, the unsteadiness parameter, and the curvature parameter. Our reports confirm that the existence of dual solutions (first and second solutions) is provable for a wide variety of control parameters within the hybrid Al_2O_3-Cu/H_2O nanofluid. An expansion in the nanoparticle volume fraction concentration can boost the skin friction coefficient and the heat transfer rate of the hybrid Al_2O_3-Cu/H_2O nanofluid in the current work. The result also implies that the hybrid nanofluid Al_2O_3-Cu/H_2O performs better in thermal conductivity than the conventional Al_2O_3-Cu/H_2O nanofluid. Meanwhile, a reduction in the unsteadiness parameter decreases the velocity gradient on the permeable shrinking slender cylinder and reduces the skin friction coefficient and local Nusselt number. Further, the new feature of the curvature parameter in the governing boundary layer system had fortified a diminution in the coefficient of skin friction and decreased heat transfer rate.

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