

Modeling and Forecasting the Realized Volatility of Bitcoin using Realized HAR-GARCH-type Models with Jumps and Inverse Leverage Effect

(Memodel dan Meramalkan Kemeruapan Nyata Bitcoin menggunakan Model Nyata Jenis HAR-GARCH dengan Lompatan dan Kesan Tuasan Songsang)

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ABSTRACT

Using the high-frequency data of Bitcoin, this study aims to model the time-varying volatility identified in the residuals of the heterogeneous autoregressive (HAR) model of realized volatility using the symmetric, asymmetric and long-memory generalized autoregressive conditional heteroscedastic (GARCH) models. We further extended these models by incorporating jumps and continuous components in the realized volatility estimators and investigating the impact of the inverse leverage effect. The Diebold Mariano and model confidence set test confirm that the forecasting performance of HAR-type models can be effectively improved by these innovations. The long memory HAR-GARCH model with jumps and continuous components provided better forecasting accuracy for Bitcoin volatility as compared to other realized volatility models. The findings of this study may benefit individual investors and risk managers who wish to minimize risks and diversify their portfolios to maximize profits in Bitcoin's investment.

Keywords: Bitcoin; HAR-GARCH; high-frequency data; inverse leverage; realized volatility

ABSTRAK

Dengan menggunakan data frekuensi tinggi Bitcoin, kajian ini bertujuan untuk memodelkan kemeruapan berbeza masa yang dikenal pasti dalam residu model autoregresi heterogen (HAR) daripada kemeruapan nyata menggunakan model simetri, asimetri dan memori panjang teritlak autoregresi bersyarat heteroskedastik (GARCH). Model-model ini terus diperluaskan dengan memasukkan lompatan dan komponen berterusan dalam penaksir kemeruapan nyata dan mengkaji kesan tuasan songsang. Diebold Mariano dan model ujian set keyakinan mengesahkan bahawa prestasi ramalan model jenis HAR dapat ditingkatkan dengan berkesan melalui inovasi ini. Model memori panjang HAR-GARCH dengan lompatan dan komponen berterusan memberikan ketepatan ramalan yang lebih baik untuk kemeruapan Bitcoin berbanding model kemeruapan nyata yang lain. Hasil kajian ini dapat memberi manfaat kepada pelabur individu dan pengurus risiko yang ingin meminimumkan risiko dan mempelbagaikan portfolio mereka untuk memaksimumkan keuntungan dalam pelaburan Bitcoin.

Kata kunci: Bitcoin; data frekuensi tinggi; HAR-GARCH; kemeruapan nyata; tuasan songsang

INTRODUCTION

Cryptocurrencies are digital decentralized currencies that rely upon cryptography for the generation, distribution, and circulation of money. Since the creation of Bitcoin, the first cryptocurrency by Satoshi Nakamoto in 2009, more than 5500 cryptocurrencies have been introduced in the market. Cryptocurrency returns are highly volatile and riskier than fiat currencies (Osterrieder

et al. 2017). The volatile market of cryptocurrencies has attracted many and its modeling and predictions have become a hot topic among researchers and financial practitioners (Zhang & Lan 2014).

Bitcoin is mainly used for investment purposes and the profitability of investments in the Bitcoin market greatly depends on the predictability of its price movements. Investment risk and uncertainty can be

minimized if a predictive model can accurately forecast the direction of the market. However, the dynamic characteristics of Bitcoin are quite complex displaying extreme observations, asymmetry, and several non-normal characteristics. It has gained a prominent place in financial markets and portfolio management, therefore, a detailed examination of its volatility dynamics is crucial for risk management.

Most of the studies on modeling the volatility of cryptocurrencies applied the popular Generalized Autoregressive Conditional Heteroscedastic (GARCH) model of Bollerslev (1986). Several GARCH models with Gaussian errors were employed by Katsiampa (2017) to model and estimate the volatility of Bitcoin. Chu et al. (2017) determined that the Integrated GARCH (IGARCH) model provided better estimates for Bitcoin's volatility than other volatility models. Liu et al. (2017) reported the superior performance of the GARCH model with Student- t errors than Gaussian and reciprocal inverse Gaussian distributions. Several univariate and multivariate GARCH models were used along with vector autoregressive specifications in the literature to understand the dynamic features of Bitcoin (Stavroyiannis & Babalos 2017). The association between Bitcoin price returns and volatility was investigated through asymmetric GARCH models by Bouri et al. (2017) while Naimy and Hayek (2018) evaluated the one-step-ahead predictive performance of both symmetric and asymmetric GARCH models with various innovations and concluded the better predictive performance of the Exponential GARCH (EGARCH) model. Ardia et al. (2019) modeled the dynamics and regime changes of Bitcoin volatility using the Markov-switching GARCH (MSGARCH) model. Zahid and Iqbal (2020) applied several stochastic volatility models to evaluate the dynamics of cryptocurrencies and the forecasting performance of a heavy-tailed stochastic volatility model was found superior.

Although the GARCH model and its extensions have been considered benchmark volatility models and are also popular among practitioners and researchers, these models treat volatility as an unobservable or latent variable and are mainly focused on daily data (Qu et al. 2018). However, daily returns are considered weak indicators of the present level of volatility. Generally, these models fail to capture the rapid volatility changes. Besides, jumps in volatility are also difficult to identify in these models (Hickey et al. 2012).

High-frequency data measure intraday observations of financial time series and provide a better understanding of dynamic characteristics of volatility as compared to daily data. Intuitively, intra-day data take into account

more information than daily data, which assists market participants to make quicker decisions. This phenomenon was first studied by Andersen and Bollerslev (1998) when they calculated the realized volatility (RV) by aggregating the squared intraday returns as the measure of ex-post daily volatility which for the first time makes volatility observable. Barndorff-Nielsen and Shephard (2002a, 2002b) provided the theoretical foundation of using RV as a proxy of the unobservable volatility based on the theory of quadratic variation which explains that RV is an unbiased ex-post estimator of daily return volatility as well as asymptotically free of measurement error under suitable conditions. Nowadays, high-frequency data are easily available due to advancements in technology and new developments in financial modeling have made it possible to directly measure and model the volatility of asset returns (Qu et al. 2018).

Corsi et al. (2003) introduced the Heterogeneous Autoregressive (HAR) model for RV. This model can capture the main empirical characteristics of financial data as well as long-range dependence. Andersen et al. (2007) incorporated the jumps and continuous components in the benchmark HAR model and constructed the generalized HAR-CJ model. They found improvements in the forecasting accuracy of the HAR-CJ model. These models have been extended and applied widely in modeling the financial volatility of returns due to their tractable estimation and competitive predictive forecasting performance (Cheong et al. 2017; Corsi et al. 2008; Haugom 2011; Qu et al. 2016).

Although realized volatility models have been around for some time, only a few studies have applied these models to model and predict the volatility of high-frequency cryptocurrencies data. Urquhart (2017) concluded that HAR model performance was better than the GARCH model in predicting the volatility of Bitcoin, though found no conclusive evidence of the leverage effect. Catania and Sandholdt (2019) illustrated that lagged jumps have no impact on the realized volatility of Bitcoin, though including the leverage component improved the volatility prediction. Yu (2019) reported that the leverage effect was a more powerful component as compared to the jump component in forecasting Bitcoin volatility using high-frequency data. Hattori (2020) analyzed the Bitcoin realized volatility by comparing different asymmetric volatility models and the results showed that EGARCH and asymmetric power ARCH (APARCH) models have higher predictability and the normal distribution fits the Bitcoin data better. Kochling et al. (2020) found evidence that the jump-robust volatility proxy based on intra-day returns and asymmetric

loss functions provide a greater distinction between equally- and out-performing GARCH-type models for the assessment of the quality of Bitcoin volatility forecast. Ahmed (2020) examined the return-volatility relation in Bitcoin markets employing high-frequency data and incorporating different RV proxies. Hung et al. (2020) found that the realized GARCH model with jump-robust realized measures can provide steady forecasts for the volatility of Bitcoin returns. Bouri et al. (2021) included a metric of US-China trade tension into an HAR model to forecast the daily RV of Bitcoin.

Nevertheless, the studies on modeling and forecasting the RV of cryptocurrencies using GARCH-type and RV models are scarce and the choice of models employed seems insufficient to provide a comprehensive analysis of different features of high-frequency cryptocurrency data. The intraday Bitcoin data have not been studied comprehensively. Hence, in this study, efforts were made to comprehensively analyze the dynamics of high-frequency Bitcoin data, to evaluate the performance of various realized volatility models in predicting the volatility of Bitcoin, and to suggest the best model for a riskier cryptocurrency, Bitcoin. To the best of our knowledge, the high-frequency data of cryptocurrencies have not been analyzed in detail as done in this study. The best model that can produce accurate and reliable forecasts for Bitcoin volatility could not only assist investors to save time and resources but make timely and better decisions. This study may help individual investors and risk managers who wish to minimize risks and diversify their portfolios and maximize profits in Bitcoin investment.

The contributions of this research can be summarized as follow: First, we combine existing HAR-type models with different variants of GARCH-type models that can capture the persistence, asymmetry, and long memory in the realized volatility of Bitcoin returns. Second, we further take an initiative and extend the HAR-GARCH-type models to investigate the effect of jumps and inverse leverage effect by the exploitation of measurement errors in realized volatility estimators. Third, the HARQ-L-GARCH-type models are then employed to investigate whether these innovations can improve the volatility forecasts of Bitcoin. These models can offer a novel perspective to model and forecast the realized volatility of Bitcoin. Finally, we employed the Diebold-Mariano test (DM) and Model Confidence Set (MCS) to robustly check the findings of the study. The DM test was applied to evaluate the pairwise forecasting performance of

competing models along with six loss functions instead of a single one as the appropriateness of a specific loss function for the evaluation of volatility models is not obvious (Lopez 2001). The MCS procedure was employed to compare the forecasting performance of more than two models at once for the selection of the best model for high-frequency Bitcoin data.

MATERIALS AND METHODS

Andersen and Bollerslev (1998) introduced a method to calculate the daily realized day t (RV_t) by aggregating the corresponding M equally spaced intra-daily returns $r_{t,j}$

$$RV_t = \sum_{j=1}^M r_{t,j}^2 \quad (1)$$

where $r_{t,j} = \log(P_{t,j})/P_{t,j-1}$, with $P_{t,j}$ being the price at day t and j is the number of observations within a day.

THE HAR-TYPE MODELS

These models were introduced by Chan et al. (2008) for modeling the intraday day (high-frequency) data. In these models, the realized variance, its continuous and jumps parts at a time t are represented by RV_t , C_t and J_t respectively.

The first model is the simple HAR model defined as (2),

$$(RV_t)^{\frac{1}{2}} = \beta_0 + \beta_1 (RV_{t-1})^{\frac{1}{2}} + \beta_2 (RV_{t-1}^w)^{\frac{1}{2}} + \epsilon_t \quad (2)$$

where $RV_{t-1}^w = (\sum_{j=1}^7 RV_{t-j})/7$ represents the realized variance of the past week.

The second model incorporates continuous and jump components and is defined as,

$$(RV_t)^{\frac{1}{2}} = \beta_0 + \beta_1 (C_{t-1})^{\frac{1}{2}} + \beta_2 (C_{t-1}^w)^{\frac{1}{2}} + \beta_3 (J_{t-1})^{\frac{1}{2}} + \beta_4 (J_{t-1}^w)^{\frac{1}{2}} + \epsilon_t, \quad (3)$$

where $RV_t^w = (\sum_{j=1}^7 RV_{t-j})/7$ is the past weekly realized variance and $J_t^w = (\sum_{j=1}^7 J_{t-j})/7$ are jump and continuous components $C_t^w = (\sum_{j=1}^7 C_{t-j})/7$ are used as explanatory variables.

In both these models, ϵ_t is assumed to be independently and identically distributed (IID) normal errors. The parameters of these models can be estimated using the most commonly used method of ordinary least squares (OLS).

THE HAR-GARCH-TYPE MODELS

The errors of the HAR-type models estimated from the OLS were found non-normal and showed volatility clustering and persistence in our empirical application with Bitcoin prices. Therefore, to model these non-normal characteristics of volatility, the residuals of (2) and (3) are decomposed as $\epsilon_t = \sqrt{h_t} e_t$, where h_t is the time-changing variance of the residuals and e_t is the IID innovation. The GARCH-type models are then applied to model h_t . The skewed-student- t distribution for $\{e_t\}$ was considered to take into account the non-normal characteristics such as skewness and high kurtosis. More specifically, we consider the basic GARCH model of Bollerslev (1986) the EGARCH model of Nelson (1991) and the FIGARCH model of Baillie et al. (1996).

The conditional variance in the GARCH model is modeled as,

$$h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1} \tag{4}$$

with $\omega > 0$, $\alpha, \beta \geq 0$ and $\alpha + \beta < 1$ guarantee weak stationarity and the positiveness of h_t .

The log conditional variance in the EGARCH model is modeled as,

$$\log(h_t) = \omega + \alpha e_{t-1} + \gamma(|e_{t-1}| - E|e_{t-1}|) + \lambda \log(h_{t-1}), \tag{5}$$

where γ is the asymmetry parameter that captures the asymmetric characteristics in Bitcoin returns.

Similarly, the FIGARCH specification models the conditional variance as

$$h_t = \omega + \lambda h_{t-1} + (1 - \lambda L - (1 - \phi L)(1 - L)^d) \epsilon_t^2 \tag{6}$$

In this way, modeling the HAR-types time-varying error variance with GARCH-type specification is called HAR-GARCH types models.

THE HARQ-L-GARCH-TYPE MODELS

Following Qu et al. (2018), we modeled the time-varying residual variance of the HAR-type models. However, these models are prone to measurement errors in any given finite sample and can suffer from the classical errors-in-variables problem. We dealt with this situation by incorporating the HARQ structure of Bollerslev et al. (2016) in the mean equation where (“Q”) represents the exploitation of the errors. Besides, since the asymmetry was observed in the RV of the Bitcoin prices (Figure 1), the HAR-types mean (2) and (3) were modified to HARQ-L-type, where “L” represents the inverse leverage. The HARQ-L model is defined as,

$$(RV_t)^{1/2} = \beta_0 + \beta_{1,t} (RV_{t-1})^{1/2} + \beta_2 (RV_{t-1}^w)^{1/2} + \beta_L (RV_{P_{t-1}})^{1/2} + \sqrt{h_t} e_t, \tag{7}$$

where $\beta_{1,t} = (\beta_1 + \beta_{1Q} (RQ_{t-1})^{1/4})$, $RV_{P_t} = RV_t \cdot I(r_t > 0)$ and $C_{P_t} = C_t \cdot I(r_t > 0)$ and $RQ_t = \frac{M}{3} \sum_{j=1}^M r_{t,j}^4$ is referred to as realized quarticity. The negative value of the coefficient β_{1Q} infers that accurately estimated realized volatility has a more significant effect on the forecasts than inadequate days having large measurement errors.

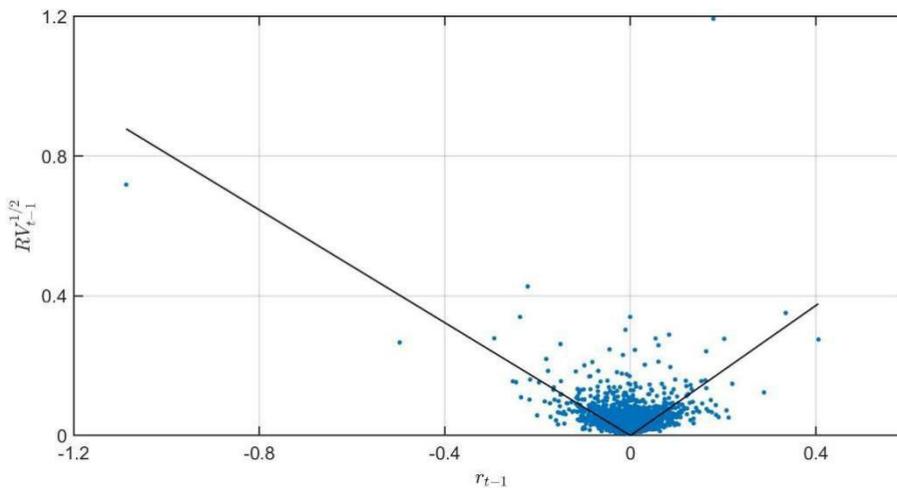


FIGURE 1. The news impact curve for the realized volatility (RV) of Bitcoin prices

Similarly, the HARQ-L-CJ model is defined as,

$$(RV_t)^{\frac{1}{2}} = \beta_0 + \beta_1(C_{t-1})^{\frac{1}{2}} + \beta_2(C_{t-1}^w)^{\frac{1}{2}} + \beta_3(J_{t-1})^{\frac{1}{2}} + \beta_4(J_{t-1}^w)^{\frac{1}{2}} + \beta_L(C_{-P_{t-1}})^{\frac{1}{2}} + \sqrt{h_t} e_t \tag{8}$$

where the possible inverse leverage effect is measured by the coefficient β_L . Again, the innovations $\{e_t\}$ are assumed to follow the skewed-student- t distribution and the GARCH-type models are used for modeling the residuals variance $\{h_t\}$. These resulting models are called HARQ-L-CJ-GARCH type models.

MODEL'S EVALUATION

The out-of-sample volatility forecasts of the models were evaluated using the pairwise forecast comparison tests of Andersen et al. (2003). Let us define (9),

$$RV_{t+1} = b_0 + b_1 \hat{h}_{Model1,t+1} + b_2 \hat{h}_{Model2,t+1} + u_{t+1}, \tag{9}$$

where RV_{t+1} represents the actual realized volatility at day $t+1$ and $\hat{h}_{Model1,t+1}$ and $\hat{h}_{Model2,t+1}$ represent its forecasts from two competing models and u_{t+1} are IID innovations. Firstly, to evaluate the usefulness of modeling the volatility of realized volatility, the HAR-type models were analyzed against the HAR-GARCH-type models. Secondly, to evaluate the value of exploiting the measurement errors in realized volatility estimators and inclusion of the inverse leverage effect, the HAR-GARCH-type models were compared against the HARQ-L-GARCH-type.

The following six average loss functions were used to better evaluate and assess the differences between the out-of-sample volatility forecasts of models:

$$MSE = \frac{1}{K} \sum_{i=1}^K (RV_t - \hat{h}_t)^2; \quad MSE_{SD} = \frac{1}{K} \sum_{i=1}^K (\sqrt{RV_t} - \sqrt{\hat{h}_t})^2;$$

$$MSE_{LOG} = \frac{1}{K} \sum_{i=1}^K (\ln RV_t - \ln \hat{h}_t)^2; \quad MAE = \frac{1}{K} \sum_{i=1}^K |RV_t - \hat{h}_t|;$$

$$MAE_{SD} = \frac{1}{K} \sum_{i=1}^K |\sqrt{RV_t} - \sqrt{\hat{h}_t}|; \quad MAE_{LOG} = \frac{1}{K} \sum_{i=1}^K |\ln RV_t - \ln \hat{h}_t|$$

In these loss functions, RV_t represents the observed realized variance at day t , \hat{h}_t is the model's predicted volatility and K is the forecast length.

To further evaluate whether two competing sets of forecasts are equally accurate, we employed the DM test by Diebold and Mariano (2002) and Hansen et al. (2011)

Model Confidence Set (MCS) procedure for comparing the predictive performance of multiple models.

DATA DESCRIPTION

This study employed the half-hourly Bitcoin price data extracted from the www.Bitcoinchart.com. The data from 28 February 2013 0:00 to 31 May 2020 0:00 (2648 days) were used. The half-hourly returns (48 per day) were calculated for every day to get $2648 \times 48 + 1 = 127,105$ half-hourly price observations. The statistical analyses were performed on R and Matlab software.

RESULTS AND DISCUSSION

EMPIRICAL ANALYSIS

The high-frequency intraday returns were demeaned using the median return $\tilde{\mu}_{t,j} = \tilde{r}_{m,d,h}$, for half-hourly data, where $\tilde{r}_{m,d,h}$, represents the median return for the t -th day in m -th month, d -th day in a week, and ($j = h$) half-hour in a day. These demeaned (returns) are used to analyze the intraday, intraweek, and periodic trends of Bitcoin prices. The intraday returns for the in-sample period were used for the calculation of the half-hourly median returns. The demeaned intraday returns $r_{t,j}^* = r_{t,j} - \tilde{\mu}_{t,j}$ were used to compute realized estimators.

Table 1 summarizes the summary statistics of the half-hourly Bitcoin prices. Large maximum, high standard deviation, positive skewness and high kurtosis indicate nonnormality in the Bitcoin prices. Table 1 also shows the summary statistics of realized measure, it's continuous and jump components. The Ljung-Box test up to the twentieth order ($Q20$) was found highly significant indicating dependence or serial correlation for all the realized measures. The Augmented Dickey Fuller (ADF) test for unit root was also found highly significant at the given confidence level for all the realized measures indicating stationarity. Hence, we employed HAR-type models and their extensions. Moreover, the jumps in the standard deviation ($J_t^{1/2}$) have a mean of 4.02%, which is 28% of the average daily realized volatility (1.11%) at the jump's detection significance level of 1%. It indicates that price jumps cannot be neglected in the Bitcoin market. Therefore, besides the simple HAR models, we applied the HAR-CJ models in this study.

TABLE 1. Summary statistics for the prices, realized volatility with its continuous and jumps part along with standard deviations for Bitcoin for the sample period from 28 February 2013 to 31 May 2020

	Mean	Min	Median	Max	SD	Skewness	Kurtosis	$Q(20)$	ADF
P_t	2481	33.25	630.20	19600	3382.9	1.7632	5.8938	2162757***	-0.5900
RV_t	0.0026	0.00002	0.0008	0.1238	0.0070	9.6657	125.99	1402***	-8.3363***
C_t	0.0004	0.00000	0.0000	0.0729	0.0071	17.0906	339.65	66.355***	-12.952***
J_t	0.0030	0.00008	0.0011	0.3258	0.0109	23.3307	657.30	2255.12***	-7.7429***
$RV_t^{1/2}$	0.0111	0.00530	0.0291	0.3518	0.0330	3.3932	18.82	5535.9***	-6.819***
$C_t^{1/2}$	0.0077	0.00000	0.0000	0.2699	0.0330	5.6731	52.00	109.75***	-11.395***
$J_t^{1/2}$	0.0402	0.00870	0.0338	0.5708	0.0330	5.0385	57.61	11697***	-6.9971***

P_t are half-hourly prices; RV_t , C_t , and J_t represent the realized volatility, continuous, and jumps segments, respectively. $Q(20)$ is the Ljung-Box statistic up to 20 lags for the error term. ADF is the Augmented Dickey-Fuller test. *** indicates a significance level at 1%. Raw high-frequency returns demeaned by a half-hour of day, day of week, and month of year

Table 2 presents the diagnostic results of the OLS estimated errors of the HAR types models. The ARCH-LM test up to 5, 10, and 20 lags were applied to test the ARCH effect. This test was found highly significant at all lags indicating that both HAR and HAR-CJ had strong autoregressive conditional heteroscedastic effects in the residual variances. We further observed positive skewness and high kurtosis in the residuals. This type of

non-normality in returns was further tested by the Jarque-Bera test which was also found significant. The highly significant Ljung-Box statistic on squared residuals (Q^2) further confirmed the long-range dependence indicating heteroscedasticity in residuals variance. Therefore, we modeled the residual variance of these models using GARCH-types models. The skewed-student- t distribution was assumed for the innovations to better model the skewness and high kurtosis observed in the residuals.

TABLE 2. Results of HAR-type models diagnostic tests for the OLS estimated error variances for the sample period from 28 February 2013 to 31 May 2020

	Skewness	Kurtosis	ARCH(5)	ARCH(10)	ARCH(20)	JB	$Q^2(20)$	$Q^2(50)$
HAR	68.00	74.769	122.1***	3.837***	34.2***	9587.3***	170.8***	438.6***
HAR-CJ	42.28	50.365	94.6***	4.06***	35.7***	105826.3***	135.1***	420.5***

ARCH(·) and JB denote the ARCH-LM test statistic and Jarque-Bera normality test statistics, respectively, while $Q^2(·)$ denotes the Ljung-Box (Q -statistics) for the squared error terms up to lag 20 and lag 50. *** denotes the significance of the test at the 1% level

IN-SAMPLE FIT RESULTS

Panel A in Table 3 presents the result of fitting the HAR model with different variants, while Panel B presents the results of the HAR-CJ model and its different variants. The coefficients β and ν are the parameters of skewed-

student- t distribution that capture the skewness and high kurtosis, respectively, while the coefficient β_L estimates the inverse leverage effect and β_{1Q} adjusts the sensitivity which quantifies the errors in the realized volatility. The values of loglikelihood (LogL), Akaike information

criteria (AIC) and Schwarz information criteria (SIC) were also reported in the table. The estimates of β were found significant with values below 1, while the estimates of the coefficient ν were also found significant with values between 2 and 3 in both panels. Hence, the assumption

of the skewed-student- t distribution for innovations seems suitable. On the other hand, all the estimates of β_{1Q} were found negative and significant (at 1% level). These findings indicate that accurately estimated realized volatility may have a significant effect on the forecasts than inadequate days having large measurement errors.

TABLE 3. Comparison of the in-sample fit results for various HAR-type models for the sample period from 28 February 2013 to 31 May 2020

Model	β_L	β_{1Q}	β	ν	LogL	AIC	SIC
<i>Panel A: HAR models</i>							
H-G	–	–	0.4982***	2.0100***	-789.0110	0.2668	0.2863
HQ-G	–	–	0.5392***	2.7077***	-83.2940	0.0398	0.0449
HQ-LG	0.0159	-0.0242***	0.4971***	2.6602***	-54.1850	0.0188	0.0294
HQ-EG	–	–	0.5451***	2.3191***	-118.1850	0.0568	0.0715
HQ-L-EG	-0.0345***	-0.0890***	0.5772***	2.2743***	-20.6780	0.0132	0.0257
HQ-FG	–	–	0.5785***	2.8266***	-78.0420	0.0324	0.0459
HQ-L-FG	0.0122	-0.0259***	0.5354***	2.7232***	-49.0120	0.0201	0.0365
<i>Panel B: HAR-CJ models</i>							
H-CJ	–	–	0.5658***	2.1372	-638.2900	0.2071	0.2215
H-CJ-G	–	–	0.5098***	2.2178	-23.4950	0.0135	0.0229
HQ-L-CJ-G	0.0059	-0.0369***	0.4865***	2.1921	-13.3858	0.0118	0.0200
H-CJ-EG	–	–	0.5364***	2.0512	-78.8900	0.0307	0.0048
HQ-L-CJ-EG	-0.0388***	-0.1221***	0.5682***	2.0338	-4.9950	0.0058	0.0187
H-CJ-FG	–	–	0.5423***	2.5458	32.5609	0.0200	0.0292
HQ-L-CJ-FG	0.0057	-0.0291***	0.5245***	2.4923	20.8920	0.0143	0.0245

H, G, EG and FG stand for HAR, GARCH, EGARCH and FIGARCH, respectively whereas Q, L and CJ stand for error measurements, inverse leverage and continuous and jump components, respectively. The coefficient and measure the leverage effect (inverse) and adjust the sensitivity in models respectively. and represent parameters of (skewed-student- t) distribution. LogL is the maximized log-likelihood value, AIC and SIC stand for the Akaike and Schwartz Information Criteria, respectively. *** denotes significance at the 1% level

For the symmetric GARCH and FIGARCH models, the estimated coefficients of β_L are found positive but not significant, whereas significant negatives values are observed for the asymmetric EGARCH specification. Such results are consistent with the inverse leverage visualized in Figure 1.

For the HAR-GARCH-type models, the LogL values were found larger and the AIC and SIC values were found smaller than the standard HAR-type model in both

panels, while the corresponding HARQ-L-GARCH type models attained larger LogL values and smaller AIC and SIC values. This indicates that models like HARQ-L-GARCH-types accomplished a better in-sample fit result as compared to the other two types of models. It is also noticed that models in Panel B attained larger LogL and smaller AIC and SIC values as compared to models in Panel A. These findings indicate the improvement in the in-sample fit when jumps are included in the volatility models.

OUT-OF-SAMPLE FORECASTS RESULTS

The out-of-sample predictive performance of the underlying models has more importance than the in-sample fit as financial decisions are mostly based on forecast results. A rolling window scheme was used to generate one-step-ahead forecasts. We selected a rolling window of size $T = 2225$ with an estimation size of $IT = 1500$ and the forecasting length $K = 725$ days. The relative forecasting power test described in (9) was employed to evaluate the forecasts for out-of-sample volatility.

Table 4 displays the findings of the pairwise forecast comparison test. It is observed from Panel A in Table 4 that the regression coefficients b_1 were all negative whereas b_2 were positive for the HAR models. The significance of b_1 coefficients at 5% level show that the

volatility forecasts improve (in most of the cases) when the HAR model's residual variance was modeled with GARCH-type specifications. Similarly, for the HAR-CJ models, the insignificance of b_1 and the significance of b_2 coefficients at 1% level show the importance of incorporating jumps and continuous components and modeling the residual variance with GARCH-type specifications. Panel B also showed that adding features like the inverse leverage effect and exploitation of error in the models such as HAR-GARCH, HAR-FIGARCH, and HAR-CJ-EGARCH improve the volatility forecasts. However, the HAR-EGARCH model was not found superior in terms of forecasting and no conclusion can be drawn from other models. This test further suggests the usefulness of the exploitation of measurement errors and incorporating the inverse leverage effect.

TABLE 4. Comparison of the out-of-sample forecasts of the HAR-type model and its different variants using relative forecasting power test for the sample period from 6 June 2018 to 31 May 2020

<i>Panel A: HAR-type v HAR-GARCH-type</i>						
	H v H-G	H v H-EG	H v H-FG	H-CJ v H-CJ-G	H-CJ v H-CJ-EG	H-CJ v H-CJ-FG
b_0 – intercept	0.199***	0.278***	0.259***	0.029	0.101	0.039
b_1 - \hat{h}_{H-type}	-1.250**	-1.668**	-1.229**	-0.159	-0.821	-0.258
b_2 - $\hat{h}_{H-G-type}$	1.175***	1.506***	1.398***	1.341***	1.989***	1.690**
<i>Panel B: HAR-GARCH-type v HARQ-L-GARCH-type</i>						
	H-G v HQ-L-G	H-E v HQ- L-EG	H-FG v HQ- L-FG	H-CJ-G v HQ- L-CJ-G	H-CJ-EG v HQ- L-CJ-EG	H-CJ-FG v HQ- L-CJ-FG
b_0 – intercept	-0.159***	0.168*	-0.199***	-0.019	-0.191***	-0.071
b_1 - $\hat{h}_{H-G-type}$	-0.212*	1.192*	-0.001	1.001	0.592*	0.642
b_2 - $\hat{h}_{HQ-L-G-type}$	-1.287***	-0.049	1.989***	0.499	1.100***	0.721

H, G, EG and FG stand for HAR, GARCH, EGARCH and FIGARCH, respectively whereas Q, L and CJ stand for error measurements, inverse leverage and continuous and jump components, respectively. *, ** and *** denote significance at 10%, 5% and 1%, respectively.

The results of commonly used loss functions along with the DM test p -values are reported in Table 5. Panel A of Table 5 illustrates that all the HAR-GARCH-type models attained smaller values of average losses than standard HAR-type models. The p -values of the DM test were all zeros for these average losses. These results (all 36 bold

average losses) indicated a significant improvement in volatility forecasts when the volatility of the realized volatility is modeled. The results in Panel B show that out of 36 losses, 21 losses were found smaller in magnitude as compared to the corresponding numbers in Panel A. Moreover, 18 out of 21 losses (in bold) had p -values of

the DM test smaller than 0.05. This showed that models like HARQ-L-GARCH-type achieved significantly better performance for volatility forecasts as compared to HAR-GARCH-type models. We also observed that a few losses (15) attained greater values as compared to the values in Panel A, but most of these values do not show the statistically significant inferior predicting performance

of the HARQ-L-GARCH-type models, since only three values (underlined in the table) are accompanied with DM test p -values smaller than 0.05. Therefore, these findings showed that the combination of leverage effect (“L”) and exploitation of error (“Q”) improve the forecasting performances of realized volatility models.

TABLE 5. Mean losses in the out-of-sample forecasts from 6 June 2018 to 31 May 2020. The Diebold Marino test -value at 10% significance level in parenthesis

<i>Panel A: HAR-type v HAR-GARCH-type</i>								
	H	H-G	H-EG	H-FG	H-CJ	H-CJ-G	H-CJ-EG	H-CJ-FG
MSE	5.968	5.474 (0.000)	5.509 (0.000)	5.517 (0.000)	5.722	5.332 (0.000)	5.351 (0.000)	5.367 (0.000)
MAE	0.615	0.556 (0.000)	0.566 (0.000)	0.558 (0.000)	0.528	0.474 (0.000)	0.485 (0.000)	0.476 (0.000)
MSE _{LOG}	0.749	0.583 (0.000)	0.608 (0.000)	0.577 (0.000)	0.688	0.672 (0.000)	0.576 (0.000)	0.544 (0.000)
MSE _{SD}	0.386	0.227 (0.000)	0.230 (0.000)	0.230 (0.000)	0.248	0.246 (0.000)	0.245 (0.000)	0.243 (0.000)
MAE _{LOG}	0.615	0.522 (0.000)	0.544 (0.000)	0.521 (0.000)	0.589	0.506 (0.000)	0.527 (0.000)	0.502 (0.000)
MAE _{SD}	0.189	0.156 (0.000)	0.162 (0.000)	0.156 (0.000)	0.158	0.126 (0.000)	0.133 (0.000)	0.126 (0.000)
<i>Panel B: HAR-GARCH-type v HARQ-L-GARCH-type</i>								
	HQ-LG	HQ-L-EG	HQ-L-FG	HQ-L-CJ-G	HQ-L-CJ-EG	HQ-L-CJ-FG		
MSE	5.376 (0.321)	6.125 (0.079)	5.480 (0.047)	5.433 (0.022)	5.581 (0.021)	5.482 (0.001)		
MAE	0.538 (0.012)	0.565 (0.472)	0.540 (0.014)	0.529 (0.360)	0.545 (0.218)	0.533 (0.001)		
MSE _{LOG}	0.546 (0.000)	0.567 (0.010)	0.539 (0.000)	0.536 (0.000)	0.551 (0.010)	0.535 (0.000)		
MSE _{SD}	0.196 (0.042)	0.219 (0.153)	0.201 (0.124)	0.196 (0.496)	0.345 (0.207)	0.200 (0.060)		
MAE _{LOG}	0.503 (0.000)	0.509 (0.000)	0.500 (0.001)	0.492 (0.000)	0.503 (0.001)	0.200 (0.060)		
MAE _{SD}	0.199 (0.000)	0.204 (0.015)	0.198 (0.002)	0.194 (0.157)	0.245 (0.240)	0.195 (0.432)		

H, G, EG and FG stand for HAR, GARCH, EGARCH and FIGARCH, respectively whereas Q, L and CJ stand for error measurements, inverse leverage and continuous and jump components, respectively. Bold numbers show innovations improve the volatility forecasts significantly. The null hypothesis of the DM test is equal forecasting performance and the significance level is 10%

The model evaluation results could differ with varying times because the RV of the Bitcoin prices shows a time-varying pattern in the evaluation period. Therefore, we further divided the forecasting period comprising 725 days into successive sub-periods of 30 days length and applied the pairwise DM test. The results were reported in Table 6. It can be seen from Panel A that the rejection frequencies in parenthesis were all close to 0. This indicates that the HAR-type models rarely performed significantly better than the HAR-GARCH-type models in sub-periods. Moreover, the HAR-GARCH-types attained significantly better predictive accuracy in about half of the sub-periods than the HAR-type models irrespective

of their basic structures like HAR-type, GARCH-type, and the specified loss functions. These results further strengthen the significance of modeling the volatility of realized variance. From Panel B of Table 6, it is noted that fewer rejection counts in parenthesis are significant. Particularly, the rejection counts were in the ranges 5 to 11 against "Q" and "L" structure, while some of the rejections outside the parenthesis ranged from 8 to 27 in favor of these structures. However, the majority of the HAR-GARCH-type models show many more rejections in favor than against these structures. It justifies that the inclusion of "Q" and "L" in RV models may provide further improvement in the Bitcoin volatility predictions.

TABLE 6. Comparison of the out-of-sample forecasts using the DM test rejection counts for the period from 6 June 2018 to 31 May 2020

<i>Panel A: HAR-type v HAR-GARCH-type</i>						
	H-G	H-EG	H-FG	H-CJ-G	H-CJ-EG	H-CJ-FG
MAE	38 (5)	33 (0)	39 (3)	30 (2)	29 (0)	29 (0)
MAE	31 (1)	34 (6)	28 (4)	34 (3)	31 (2)	34 (1)
MSE _{LOG}	39 (2)	38 (1)	37 (4)	34 (1)	34 (0)	34 (0)
MSE _{SD}	36 (3)	41 (1)	37 (4)	35(1)	33 (0)	34 (0)
MAE _{LOG}	40 (5)	39 (5)	39 (4)	35 (2)	35 (2)	35 (4)
MAE _{SD}	37 (5)	36 (6)	39 (6)	36 (3)	34 (2)	35 (3)
<i>Panel B: HAR-GARCH-type v HARQ-L-GARCH type</i>						
	HQ-L-G	HQ-L-EG	HQ-L-FG	HQ-L-CJ-G	HQ-L-CJ-EG	HQ-L-CJ-FG
MSE	24 (7)	16 (7)	13 (8)	7 (9)	8 (10)	19 (8)
MAE	13 (6)	13 (7)	21 (7)	10 (8)	12 (8)	16 (8)
MSE _{LOG}	21 (5)	17 (8)	19 (7)	16 (7)	12 (11)	18 (6)
MSE _{SD}	18 (7)	11 (9)	24 (8)	15 (8)	8 (11)	15 (8)
MAE _{LOG}	27 (8)	19 (9)	15 (8)	19 (8)	16 (9)	20 (8)
MAE _{SD}	26 (8)	17 (8)	21 (8)	16 (9)	17 (9)	23 (8)

H, G, EG, and FG stand for HAR, GARCH, EGARCH, and FIGARCH, respectively, whereas Q, L and CJ stand for error measurements, inverse leverage and continuous and jump components, respectively. The rejections in favor of innovations (values outside parenthesis) against the rejections not in favor of innovations (values inside the parenthesis) at 10% significance level

Figure 2 illustrates the results of the MCS procedure on the out-of-sample forecasting performance of all the models. The p -values > 0.1 specify that the compatible

models such as the HARQ-L-CJ-GARCH and the HARQ-L-CJ-FIGARCH models were in $\hat{M}_{0.9}^*$ according to the specified loss functions. Therefore, we can say

that these two models are the superior models for the volatility forecasts of Bitcoin returns. Furthermore, models like the HAR-CJ-GARCH, HAR-CJ-FIGARCH, and HARQ-L-FIGARCH were in $\hat{M}_{0,9}^*$ according to the loss functions, therefore, are declared favorable models in this study. Moreover, the application of the GARCH and the FIGARCH specifications for modeling the realized

volatility was found more suitable as compared to the EGARCH structure. These findings showed that HAR-CJ based models (models 8-14) have more p -values > 0.1 as compared to the HAR based (models 1-7). It further justified that dividing the RV into continuous and jumps components enhanced the volatility forecast of Bitcoin. These results are in accordance with the in-sample fit and further confirmed the results of Table 5.

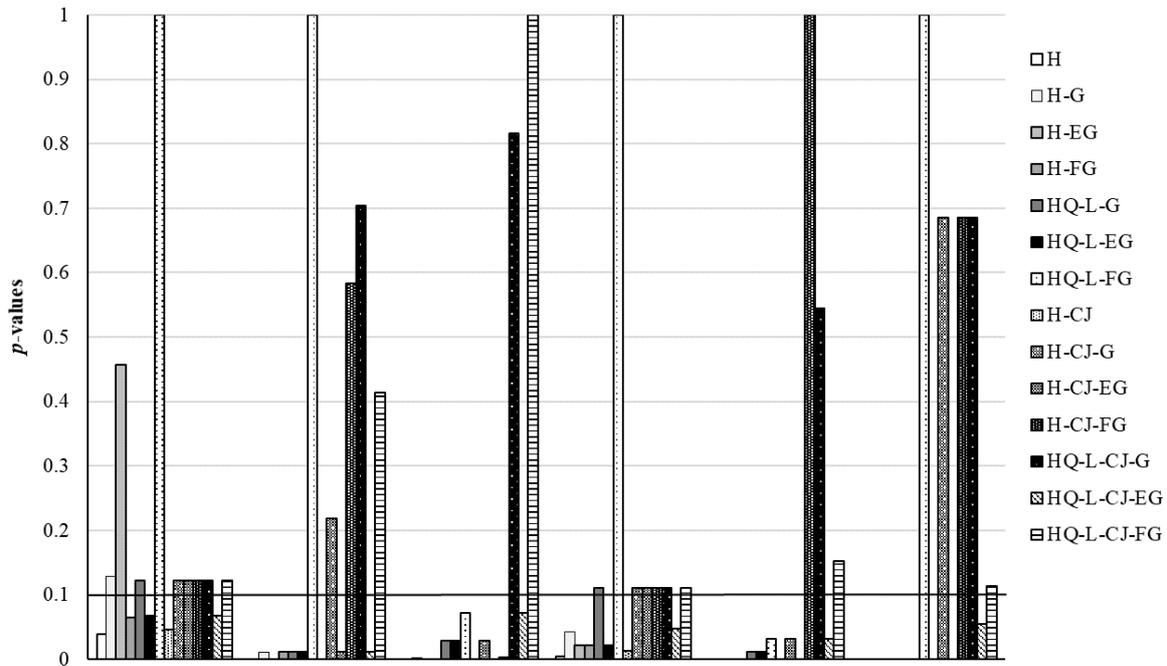


FIGURE 2. Comparison of the out-of-sample forecast by MCS test -values for all the models results for the period from 6 June 2018 to 31 May 2020. H, G, EG and FG stand for HAR, GARCH, EGARCH and FIGARCH, respectively, whereas Q, L and CJ stand for error measurements, inverse leverage and continuous and jump components, respectively. Bold horizontal line represent p -value = 0.1

The MCS test was also applied by considering a sub-period of successive 30-day duration since the difference in the results of the model comparison can occur as time varies. The significance level is set to 10% and the results were shown in Figure 3. The height of the bars represent the persistence/survival counts in $\hat{M}_{0,9}^*$ of all the prescribed models. The leading survival frequency in each loss function was marked with a ★. It was concluded from the figure that the survival frequencies of the HAR-GARCH and HAR-FIGARCH models are much greater than the benchmark HAR model. Similarly, the HAR-CJ-GARCH and HAR-CJ-FIGARCH models have much greater survival frequencies than the HAR-CJ model whereas the survival frequencies for HAR-EGARCH models were found slightly greater than the standard HAR model.

Finally, the survival frequencies for the HAR-CJ-EGARCH model were not found higher than the HAR-CJ model.

These results indicate that whether the jumps are included in RV models or not, modeling the variance of RV with models such as GARCH and the FIGARCH specifications successfully improves the out-of-sample forecast, though it was not found true for the EGARCH specification. Hence, the GARCH and FIGARCH specifications may be considered more appropriate for modeling the volatility of RV than the EGARCH. These results were consistent with the results presented in Figure 2. Moreover, the MCS procedure showed that survival counts for "Q" and "L" structures are mostly larger as compared to their standard models. It also showed that including these features in RV estimators successfully

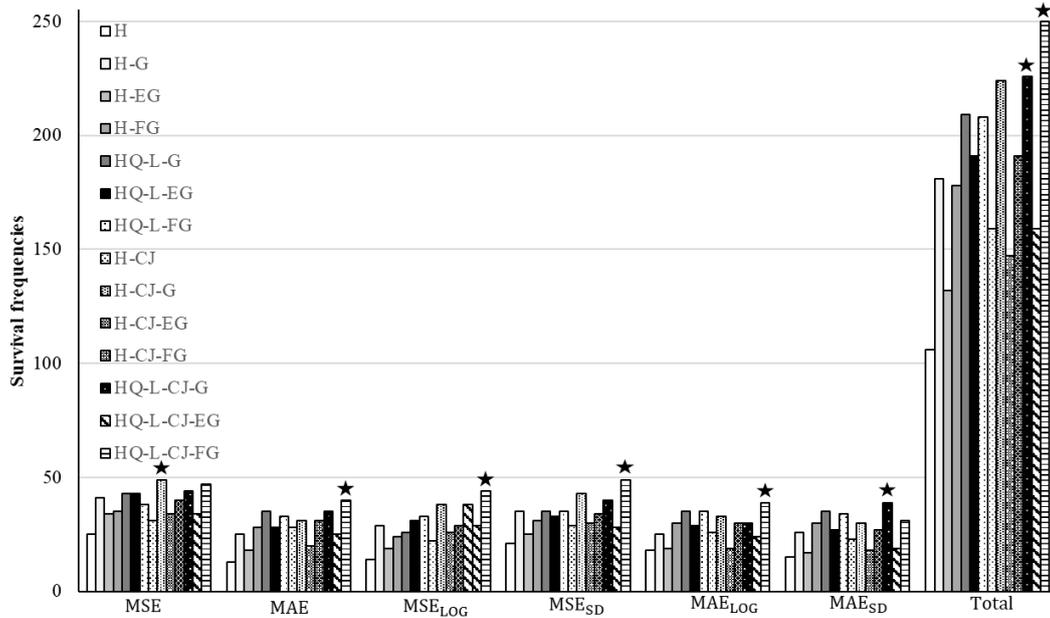


FIGURE 3. Comparison of the out of sample forecast result by survival frequencies in the model confidence set for all the models for the period from 6 June 2018 to 31 May 2020. H, G, EG and FG stand for HAR, GARCH, EGARCH and FIGARCH, respectively whereas Q, L and CJ stand for error measurements, inverse leverage and continuous and jump components, respectively. Significance level at 10% for the MCS. ★ indicates the leading survival frequencies with respect to prescribed loss functions

enhanced volatility forecasts of the HAR-GARCH-type models. The HAR-CJ structure leads to larger survival counts indicating that dividing the RV into continuous and jumps components enhanced volatility forecasts of Bitcoin returns. The total survival frequencies of the HAR-GARCH and HAR-FIGARCH models were 181 and 178, respectively, while the total survival frequencies for the HAR-CJ model were 159. These results showed that the improvement in forecasting performance by including jumps are less significant than properly modeling the error variance. Finally, the models like HARQL-CJ-FIGARCH and HARQ-L-CJ-GARCH were found high in ranking among all other competing models as they attained the largest (250) and second-largest (226) survival frequencies. For these two models, the survival counts range was from 31 to 49 and 30 to 44, respectively, indicating the HARQL-CJ-FIGARCH model as the best performing model for the Bitcoin volatility forecast in this study.

Figure 4 shows the realized volatility forecasts from the HAR and HARQ-L-CJ-FIGARCH models along with the observed realized variance from 6 June 2018 to 31 May 2020 (725 trading days). It was evident that the forecasted realized volatilities are close to the observed realized

variance. Besides, the HARQ-L-CJ-FIGARCH provides higher forecast accuracy as compared to the HAR model. These findings further confirmed the tabulated results presented earlier.

Few past studies have analyzed the high-frequency data of Bitcoin. Urquhart (2017) and Yu (2019) compared the HAR and GARCH models whereas Kochling et al. (2020) and Hattori (2020) focused on GARCH-type models to forecast the RV of Bitcoin. Ahmed (2020) proxied the intraday Bitcoin price variability by four different measures. However, the present study applied innovative HAR-type models combined with different GARCH specifications for better modeling and forecasting of the volatility of Bitcoin. Our results suggested that decomposing the total realized volatility into jumps and the continuous components provided improved forecasts. Our findings are somewhat consistent with Yu (2019) who declared the best HAR models having leverage and jumps components. Additionally, this study exploited the measurement errors and investigated both the jumps and inverse leverage effect in the HAR-GARCH-type models. Similar to the present study, Catania and Sandhold (2019) illustrated that the inclusion of the leverage component improved the performance of the

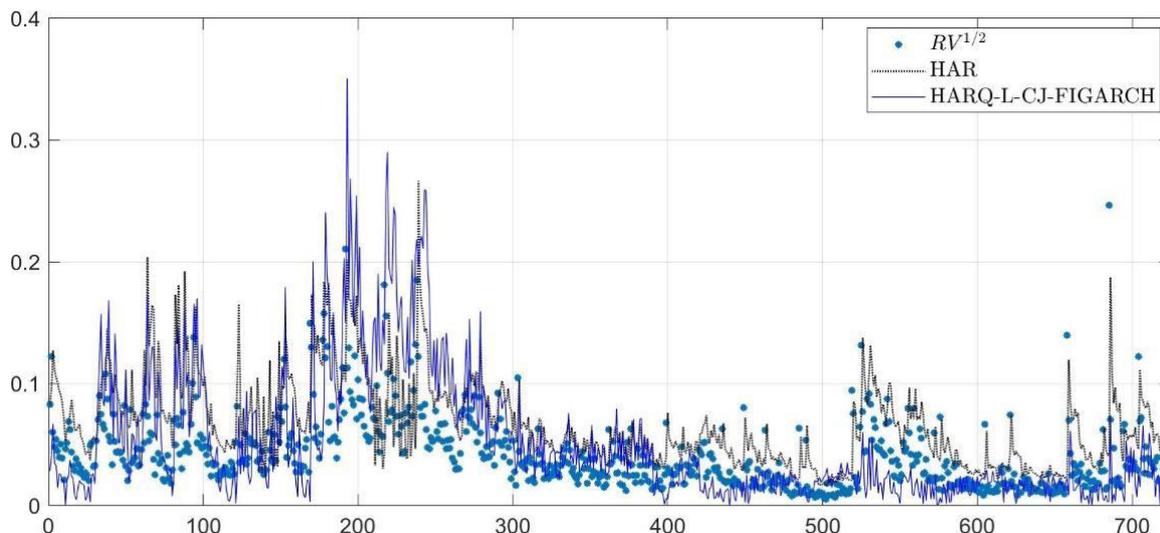


FIGURE 4. Comparison of out-of-sample realized volatility forecasts with observed realized variance from 6 June 2018 to 31 May 2020

benchmark RV models. However, the inclusion of jumps did not provide better predictions in their study all the time. Different from their study, Our findings showed that the combination of HAR-GARCH-type models with jumps and inverse leverage effect improved the accuracy of volatility forecasts of Bitcoin as compared to the simple HAR or GARCH-type models. Our findings are also in line with Ahmed (2020) and Kochling et al. (2020) who showed that the inclusion of jumps improves the volatility forecasts.

CONCLUSION

In this article, an effort was made to model and forecast the Bitcoin volatility using high-frequency prices. The volatility of realized volatility was modeled with various GARCH-type models. We then extended the HAR-GARCH-type models to the HARQ-L-CJ-GARCH-type models by exploiting the measurement error in the realized volatility estimators and investigated the effect of inverse leverage and jump. Our findings showed that the latter models significantly outperformed the former in in-sample fit. Besides, we also found an improvement in the in-sample fit when jumps are included in the volatility models. The DM test confirmed the superior out-of-sample forecasting performance of HARQ-L-GARCH-type models under various loss functions. The MCS test showed that the forecasting performance of the HAR-type models can be effectively improved by

modeling the time-varying volatility of the residual with GARCH-type models. Similarly, the inclusion of measurement errors and leverage effect in HAR-GARCH-types models further improved the predictive accuracy. The performance of HARQ-L-GARCH-type models was found better than HAR-GARCH-type models. Moreover, we found an improvement in the out-of-sample forecasts when jumps were included in the volatility models. Finally, the HARQ-L-CJ-FIGARCH model attained superior forecasting accuracy for Bitcoin volatility as compared to other competing models validating the importance of inclusion of jumps, inverse leverage effect, and exploiting the measurement error in the realized volatility estimators. The findings of this study may benefit individual investors and risk managers who wish to minimize risks and diversify their portfolios to maximize profits in Bitcoin's investment.

REFERENCES

- Ahmed, W.M.A. 2020. Is there a risk-return trade-off in cryptocurrency markets? The case of bitcoin. *Journal of Economics and Business* 108: 1-21.
- Andersen, T.G. & Bollerslev, T. 1998. Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International Economic Review* 39(4): 885-905.
- Andersen, T.G., Bollerslev, T. & Diebold, F.X. 2007. Roughing it up: Including jump components in the measurement, modeling and forecasting of return volatility. *The Review of Economics and Statistics* 89(4): 701-720.

- Andersen, T.G., Bollerslev, T., Diebold, F.X. & Labys, P. 2003. Modeling and forecasting realized volatility. *Econometrica* 71(2): 579-625.
- Ardia, D., Bluteau, K. & Rüede, M. 2019. Regime changes in Bitcoin GARCH volatility dynamics. *Finance Research Letters* 29: 266-271.
- Baillie, R.T., Bollerslev, T. & Mikkelsen, H.O. 1996. Fractionally integrated generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics* 74(1): 3-30.
- Barndorff-Nielsen, O.E. & Shephard, N. 2002a. Econometric analysis of realized volatility and its use in estimating stochastic volatility models. *Journal of the Royal Statistical Society. Series B (Statistical Methodology)* 64(2): 253-280.
- Barndorff-Nielsen, O.E. & Shephard, N. 2002b. Estimating quadratic variation using realized variance. *Journal of Applied Econometrics* 17(5): 457-477.
- Bollerslev, T. 1986. Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics* 31(3): 307-327.
- Bollerslev, T., Patton, A.J. & Quaedvlieg, R. 2016. Exploiting the errors: A simple approach for improved volatility forecasting. *Journal of Econometrics* 192(1): 1-18.
- Bouri, E., Azzi, G. & Dyrhberg, A.H. 2017. On the return-volatility relationship in the bitcoin market around the price crash of 2013. *Economics* 11: 1- 17.
- Bouri, E., Gkillas, K., Gupta, R. & Pierdzioch, C. 2021. Forecasting relaiized volatility of bitcoin: The role of the trade war. *Computational Economics* 57(1): 29-53.
- Catania, L. & Sandholdt, M. 2019. Bitcoin at high frequency. *Journal of Risk and Financial Management* 12(36): 1-20.
- Cheong, C.W., Cherng, L.M., Isa, N.M. & Hoong, P.K. 2017. The HARX-GJR-GARCH skewed-t multipower realized volatility modelling for S&P 500. *Sains Malaysiana* 46(1): 107-116.
- Chu, J.S., Chan, S., Nadarajah, S. & Osterrieder, J. 2017. GARCH modeling of cryptocurrency. *Journal of Risk and Financial Management* 10(4): 1-15.
- Chan, K.F., Gray, P. & Campen, B.Y. 2008. A new approach to characterizing and forecasting electricity price volatility. *International Journal of Forecasting* 24(4): 728-743.
- Corsi, F., Mittnik, S., Pigorsch, C. & Pigorsch, U. 2008. The volatility of realized volatility. *Econometric Review* 27(1-3): 46-78.
- Corsi, F., Zumbach, G., Muller, U.A. & Dacorogna, M.M. 2003. Consistent high-precision volatility from high-frequency data. *Economic Notes* 30(2): 183-204.
- Diebold, F.X. & Mariano, R.S. 2002. Comparing predictive accuracy. *Journal of Business & Economic Statistics* 20(1): 134-144.
- Hansen, P.R., Lunda, A. & Nason, J.M. 2011. The model confidence test. *Econometrica* 79(2): 453-497.
- Haugom, E. 2011. Some stylized facts about high-frequency Nord Pool forward electricity prices. *Journal of Energy Markets* 4(1): 21-49.
- Hattori, T. 2020. A forecast comparison of volatility models using realized volatility: Evidence from the bitcoin market. *Applied Economics Letters* 27(7): 591-595.
- Hickey, E., Loomis, D.G. & Mohammadi, H. 2012. Forecasting hourly electricity prices using ARIMAX-GARCH models: An application to MISO hubs. *Energy Economics* 34(1): 307-315.
- Hung, J.C., Liu, H.C. & Yang, J.J. 2020. Improving the realized GARCH's volatility forecast for bitcoin with jump-robust estimators. *The North American Journal of Economics and Finance* 52(101165): 1-11.
- Katsiampa, P. 2017. Volatility estimation for bitcoin: A comparison of GARCH model. *Economics Letters* 158: 3-6.
- Kochling, G., Schmidtke, P. & Posch, P.N. 2020. Volatility forecasting accuracy for bitcoin. *Economics Letters* 191(108836): 1-17.
- Liu, R., Shao, Z., Wei, G. & Wang, W. 2017. GARCH model with fat-tailed distributions and bitcoin exchange rate returns. *Journal of Accounting Business and Finance Research* 1(1): 71-75.
- Lopez, J.A. 2001. Evaluating the predictive accuracy of volatility models. *Journal of Forecasting* 20(2): 87-109.
- Naimy, V.Y. & Hayek, M.R. 2018. Modelling and predicting the bitcoin volatility using GARCH models. *International Journal of Mathematical Modelling and Numerical Optimisation* 8(3): 197-215.
- Nelson, D.B. 1991. Conditional heteroskedasticity in asset returns: A new approach. *Econometrica* 59(2): 347-370.
- Osterrieder, J., Lorenz, J. & Strika, M. 2017. Bitcoin and cryptocurrencies- not for the faint-hearted. *International Finance and Banking* 4(1): 56-94.
- Qu, H., Chen, W., Niu, M. & Li, X. 2016. Forecasting realized volatility in electricity markets using logistic smooth transition heterogeneous autoregressive models. *Energy Economics* 54: 68-76.
- Qu, H., Duan, Q. & Niu, M. 2018. Modeling the volatility of realized volatility to improve volatility forecasts in electricity markets. *Energy Economics* 74: 767-776.
- Stavroyiannis, S. & Babalos, V. 2017. Dynamic properties of the Bitcoin and the US market. *Social Science Research Network* <http://dx.doi.org/10.2139/ssrn.2966998>.
- Urquhart, A. 2017. The volatility of bitcoin. *Social Science Research Network* <http://dx.doi.org/10.2139/ssrn.2921082>.
- Yu, M. 2019. Forecasting bitcoin volatility: The role of leverage effect and uncertainty. *Physica A: Statistical Mechanics and its Applications* 553(120707): 1-22.
- Zahid, M. & Iqbal, F. 2020. Modeling the volatility of cryptocurrencies: An empirical application of stochastic volatility models. *Sains Malaysiana* 49(3): 703-712.
- Zhang, H. & Lan, Q. 2014. GARCH-type model with continuous and jump variation for stock volatility and its empirical study in China. *Mathematical Problems in Engineering* (Special Issue: Nonlinear Problems: Mathematical Modeling, Analyzing, and Computing for Finance). 2014: Article ID. 386721.

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