Sains Malaysiana 54(6)(2025): 1617-1628
*http://doi.org/10.17576/jsm-2025-5406-16*

# Detection Procedure of Structural Changes in State-Space Models: Impulse and Steps Indicator Saturation Technique

(Prosedur Pengesanan Perubahan Struktur dalam Model Ruang-Keadaan: Teknik Ketepuan Penunjuk Impuls dan Langkah)

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*Received: 18 July 2024/Accepted: 13 March 2025*

ABSTRACT

The presence of extreme structural change in a sequence of data points over time may have a detrimental impact on the estimation of economic and financial indicators. Anomalies caused by these extreme values can distort the estimated parameters, diminish the accuracy of the time series model, and potentially lead to inaccurate forecasts. In this research, a general-to-specific modeling approach is utilized to identify the structural changes through indicator saturation within the framework of a state-space models as an alternative to current method. By focusing on impulse and steps indicator saturation, this study evaluates their effectiveness through Monte Carlo simulations that are replicated 1000 times. The Monte Carlo experiments demonstrate that the efficiency of indicator saturation is heavily dependent on factors such as the magnitude of the structural change, the level of statistical significance, and the position of an extreme value within the series. Subsequently, this study employs the combined impulse and steps indicator saturation to detect structural breaks in the FTSE 100 daily closed stock price index. The most important findings relate to the coefficients for the structural breaks at $t = 2020.M1$ and $t = 2020.M3$ are estimated at $\hat{\beta}_{t=2020.M1} = 0.16$ and $\hat{\beta}_{t=2020.M3} = -0.29$, respectively. The findings show that the characteristics, position, and direction of the extreme values detected by impulse indicator saturation coincide with the occurrence of the COVID-19 pandemic, which has had a global impact on economic activities. This finding may lead to better understanding of how the stock markets in UK reacts to government policy due to the COVID-19 pandemic.

Keywords: General-to-specific; indicator saturation; model selection; Monte Carlo; structural changes

ABSTRAK

Kehadiran perubahan struktur yang ketara dalam satu data siri masa boleh memberi kesan buruk terhadap penganggaran penunjuk ekonomi dan kewangan. Nilai ekstrem yang menyebabkan anomali boleh memesongkan parameter yang dianggarkan, mengurangkan ketepatan model siri masa dan berpotensi menghasilkan ramalan yang tidak tepat. Dalam kajian ini, pendekatan pemodelan umum kepada khusus digunakan untuk mengenal pasti perubahan struktur melalui petunjuk ketepuan dalam kerangka model ruang keadaan sebagai alternatif kepada kaedah semasa. Dengan memberi tumpuan kepada petunjuk ketepuan impuls dan langkah, kajian ini menilai keberkesanannya melalui simulasi Monte Carlo yang diulang sebanyak 1000 kali. Uji kaji Monte Carlo menunjukkan bahawa kecekapan ketepuan penunjuk sangat bergantung kepada faktor seperti magnitud perubahan struktur, tahap signifikan statistik dan kedudukan nilai ekstrem dalam siri data tersebut. Seterusnya, kajian ini menggunakan gabungan ketepuan penunjuk impuls dan langkah untuk mengesan perubahan struktur dalam indeks harga penutup harian FTSE 100. Penemuan paling penting berkaitan dengan pekali bagi perubahan struktur pada penemuan ini menunjukkan bahawa ciri, kedudukan dan arah nilai ekstrem yang dikesan oleh ketepuan penunjuk impuls bertepatan dengan berlakunya pandemik COVID-19, yang telah memberi kesan global terhadap aktiviti ekonomi. Penemuan ini boleh membawa kepada pemahaman yang lebih baik tentang bagaimana pasaran saham di UK bertindak balas terhadap dasar kerajaan akibat pandemik COVID-19.

Kata kunci: Monte Carlo; pemilihan model; perubahan struktur; petunjuk ketepuan; umum kepada tertentu

## INTRODUCTION

Time series data are predominantly observational in nature. Structural changes within time series can significantly impact the estimation of models, primarily employed for economic and financial indicators. Outlier and structural breaks are the example of the structural changes in time series data. The sources structural changes may stem from extraordinary events such as warfare, pandemics, policy modifications, or natural disasters (Castle, Hendry & Martinez 2023). The presence of such structural changes consistently raises critical concerns regarding the accuracy and efficiency of model parameter estimations. Failing to account for these outliers or structural breaks can result in the misclassification of a distribution as fat-tailed when it is a thin-tailed distribution. Furthermore, Castle and Hendry (2019) demonstrated that neglecting existing structural changes in models leads to excessively wide interval forecasts. Unanticipated events can trigger forecast failures, causing actual outcomes to deviate significantly from forecasted values, measured in standard errors. These forecast failures are often attributed to location shifts in the time series data after the forecasts have been made. The implications of forecast failure extend to decision-making and policy formation, potentially leading to poor decisions and misguided policies.

Numerous methods available in the literature including statistical, regression based and machine learning approaches. Statistical based techniques such as Z-score, modified Z-score and interquartile range require sufficient information on the dataset. The application of outlier detection algorithm has been extensively explored in real applications such as in engineering, oil and gas, fraud detection and image processing (Alimohammadi & Nancy Chen 2022; Huang, Cheng & Zhang 2023; Sewwandi, Li & Zhang 2024; Shao et al. 2022; Yu et al. 2014). Machine learning techniques, like classification algorithms, detect outliers by identifying data outside the decision boundary (Bergman & Hoshen 2020) with recent study by Vishwakarma, Paul and Elsawah (2020) exploring neural network algorithm to detect outliers in univariate time series. Another study conducted by Uzabaci, Ercan and Alpu (2020) proposed two machine learning techniques to identify outliers viz blocked adaptive computationally efficient outlier nominators (BACON) and fast minimum covariance determinant (FAST-MCD) algorithms for multivariate datasets. However, these methods primarily identify outlier locations and often fail to estimate model parameters accurately in the presence of outliers. Additionally, machine learning algorithms are computationally expensive, require minimum thresholds for dataset size or distance, and perform poorly with small datasets (Alimohammadi & Nancy Chen 2022). To address these limitations, we propose using the indicator saturation technique within a GETS framework, as supported by recent studies on its application to detect outliers and structural breaks (Castle & Kurita 2021; Castle, Doornik & Hendry 2012, 2011; Pellini 2021).

Impulse indicator saturation (IIS) and step indicator saturation (SIS) are the two types of indicator saturation technique proposed by Hendry (1999). Both types involve adding a binary set of dummy variables each corresponding to a single observation, and including these indicators in a regression model. The performance of the IIS and SIS were excellent when applied to basic structural time series. However, this particular analysis has only been conducted by Marczak and Proietti (2016). Many issues highlighted here remain unexplored in the literature about the performance of indicator saturation in state space models. The complete potential of this approach has yet to be confirmed; therefore, our attention is directed towards the local level model (LLM) including the slope (LLTM) and seasonal component (LLTSM). State-space models provide a robust framework for detecting outliers and structural breaks, including through methods like IIS and SIS. These models are characterized by random disturbances around an underlying level, making them ideal for analysing fluctuations and identifying deviations. While the standard approach relies on auxiliary residuals for detection (Durbin & Koopman 2012), it requires prior information about the location of potential outliers or breaks. By integrating IIS and SIS with state-space methods, we aim to overcome this limitation and improve detection accuracy.

This study is conducted to immediately detect outliers or structural breaks in a series to avoid misspecification of estimation and distortion of forecast accuracy. Specifically, combination of IIS and SIS are used to detect structural change when they are near to the forecast origin. This is done by applying the IIS proposed by Hendry (1999) to identify the unknown amount, location, and magnitude of outliers in the series examined. IIS works by annexing a set of dummy variables as an intervention for each observation in the series. A plethora of studies on this approach can be found in Castle, Doornik and Hendry (2012, 2011), Castle et al. (2015), Johansen and Nielsen (2009), Marczak and Proietti (2016), and Santos, Hendry and Johansen (2008), using *Autometrics* embodied in *OxMetrics* as a computational tool to perform GETS algorithm. Although similar approach was used by Marczak and Proietti (2016), this work differs in the way it handles the dummy indicators in state-space models by combining impulse and steps indicators in the model estimation. To the best of our knowledge, the performance of IIS+SIS integrated with the state-space models has not been scrutinized yet. This combination offers several advantages over existing methods. Unlike traditional approaches that require prior information about the location of outliers or structural breaks, IIS and SIS systematically test for these structural changes across all potential time points, making the process more robust and automated. Hence, this study is one of the first attempts to thoroughly examine the performance of IIS and SIS in the context of the state-space models by assessing the potency and gauge values. Further, we apply the IIS+SIS to detect any structural change in stock price series, specifically FTSE 100.

The subsequent part of this investigation is organized as follows. In the methodology section, an in-depth examination of the structure for the state-space models in outlier detection is provided, along with the presentation of the indicator saturation concepts within the state-space framework. Next, the section commences by outlining the simulation settings for the Monte Carlo experiment, followed by a summary of the performance of Monte Carlo simulations on the detection power of IIS+SIS. In the empirical applications section, IIS and SIS are subsequently employed to the actual stock price data to identify outlier or structural breaks. Last section brings the paper to a close.

## METHODOLOGY

This section summarized the whole procedure of outlier detection algorithm in state-space framework as illustrated in Figure 1. Firstly, the procedure started with generating an artificial time series dataset based on the state-space models. Then, the series was contaminated with the additive outlier (AO) for performance evaluation purposes. This is followed by the Monte Carlo simulation experiments replicated at M = 1000 times. The performance of indicator saturation was measured using potency and gauge metrics. Finally, the application of IIS and SIS to a real dataset to capture the outlier in stock price data.

## STATE-SPACE MODELS

The simplest form of state space model is local level model (LLM). The model consists of level component which varies over time. The level component acts as an intercept in the classical regression model. The LLM can be formulated as

$$y_t = \mu_t + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2) \tag{1}$$

$$\mu_{t+1} = \mu_t + \omega_t, \quad \omega_t \sim NID(0, \sigma_\omega^2) \tag{2}$$

for $t = 1,2,\ldots,T$ where $\mu_t$ is the unobserved level component at time *t*, $\varepsilon_t$ is the irregular component at time *t*, and $\omega_t$ is the level disturbance at time *t*. The $\varepsilon_t$ and $\omega_t$ are all assumed to be independent and identically distributed with zero mean and variances $\sigma_\varepsilon^2$ and $\sigma_\omega^2$, respectively. Equation (1) is defined as the observation equation and Equation (2) is defined as the transition state equation. The transition equation shows the fundamental values based on a random walk. The component $\varepsilon_t$ is defined as noise and assumed to be independent and identically distributed. In this study, we define the signal-to-noise ratio as $q = \sigma_\varepsilon^2/\sigma_\xi^2$. Thus, the local level model also can be referred to as the *random walk plus noise* model (Harvey & Koopman 2000).

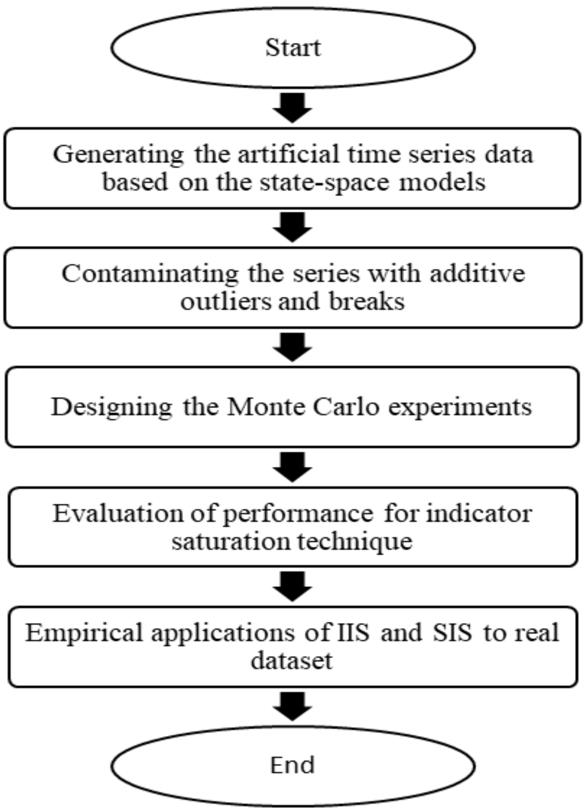


FIGURE 1. Summary of the performance evaluation and application of indicator saturation

Local linear trend model is an extension of LLM, by adding the slope component, $v_t$ to the local level model. In this model, both level and slope vary over time. Hence, the local linear trend model (LLTM) is formulated as follows:

$$y_t = \mu_t + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2) \tag{3}$$

$$\mu_{t+1} = \mu_t + \nu_t + \omega_t, \qquad \omega_t \sim NID(0, \sigma_\omega^2) \tag{4}$$

$$\nu_{t+1} = \nu_t + \psi_t, \qquad \omega_t \sim NID(0, \sigma_\psi^2) \tag{5}$$

where t = 1, 2, …, T; $\omega_t$ and $\psi_t$ are independent and identically distributed. With equation (3) as the observation equation, Equations (4) and (5) represent the state equations. In particular, $\mu_t$ remains as the trend component, and $v_t$ as the slope of trend component, differs from the slope of classic regression line. Therefore, $v_t$ cannot be conceived as the coefficient of regressors in linear regression model when $\omega_t$ and $\psi_t$ equal to zero.

Seasonal effect refers to the recurring pattern in time series within a specific period of time. In state-space models, seasonal effect can be modelled by adding the seasonal component, either in LLM or LLTM. Then, it can be formulated as follows; for simplicity, the seasonal component of $\phi_t$ is added to LLM:

$$y_t = \mu_t + \phi_t + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{6}$$

$$\mu_{t+1} = \mu_t + \delta_t, \qquad \delta_t \sim N\left(0, \sigma_\delta^2\right) \tag{7}$$

$$\phi_{t+1} = \sum_{j=1}^{s-1} \phi_{t+1-j} + \omega_t \,, \, \omega_t \sim N(0, \sigma_\omega^2) \tag{8}$$

where t = 1, 2, …, T; $\delta_t$ and $\omega_t$ are independent and identically distributed. Equation (6) defined as the observation equation, Equations (7) and (8) represent the state equations. The addition of error term, $\omega_t$ allows the variation of seasonal component over time. The seasonal pattern can be modelled for a specific period of time ($s$ = 7 for weekly data; $s$ = 12 for monthly data; $s$ = 4 for quarterly data); let $s$ as the months in year $i$ and $j$ as a specific month.

## DETECTION PROCEDURE USING INDICATOR SATURATION (IS)

The detection procedure using indicator saturation technique are summarized here. It is worth mentioning that the selection of retained indicator can be achieved using sequential and non-sequential selection algorithms. The sequential selection, which involves the elimination of one insignificant indicator at a time at a specific level of significance. On the other hand, non-sequential selection is another selection method that eliminates all insignificant indicators simultaneously in every block at a specific level of significance. This has been implemented using the R software package. In summary, the procedure takes the following form when indicators are added to $T$ observations: 1) The indicators are divided into $b$ = 2 blocks, $b_1$ and $b_2$; 2) Estimation of parameters including the impulse indicators for both blocks using ordinary least square (OLS) method as shown in Figure 2(a) and Figure 2(c); 3) Selection of retained indicators using sequential or non-sequential approach at chosen significance level as presented in Figure 2(b) and Figure 2(d); 4) Run the model selection using GETS approach to obtain the terminal model in the first block; 5)

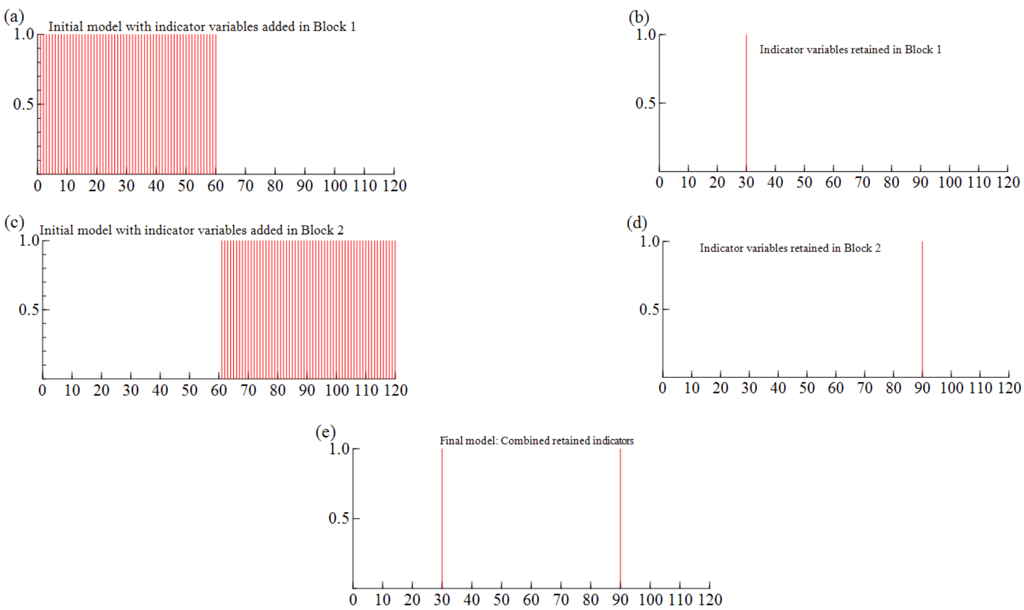


FIGURE 2. The impulse indicator saturation algorithm depicted when the additive outlier located at $t = 30$ and $t = 90$

Recommence steps 2 to 4 for second block; 6) Combine all the significant indicators retained from both blocks; and 7) Rerun the model selection on retained indicators to obtain the final terminal model as depicted in Figure 2(e). Details procedure of IIS and SIS techniques are presented in Castle et al. (2015) and Che Rose, Ismail and Tumin (2021).

## MONTE CARLO SIMULATION EXPERIMENTS

The performance of the IIS and SIS approach were assessed through Monte Carlo experiments. Various alternative settings are considered to examine the procedure's robustness. Each experiment consists of *M = 1000* replications. The following are specifications for the simulation settings for a reference data generating process (DGP): a) Sample size *T = 120* observations were generated based on the state-space models; b) One additive outlier (AO) is located at the middle of the sample. Meanwhile, two AO's were predetermined at the [0.25, 0.75] points as a proportion of observations, *T*; c) Target size or significance level, α = 0.1%, 1% and 2.5%. According to Mariscal and Powell (2014), these values will determine the statistical tolerance of the procedure. For example, a target of 0.01 for IIS indicates that on average, we accept 1 impulse dummy that may not be in the data generating process for every 100 observations; d) We labelled the magnitude of an AO as *zσ* where *z* is a positive integer. Meanwhile, σ is the prediction error standard deviation (PESD) of the series. The magnitude of AO varies between 3σ, 5σ, 7σ, 9σ, and 12σ; e) We apply the block-splitting algorithm by partitioning the indicator variables into six blocks to lower the variance of estimates; and f) The location of AO also varies based on the share of the sample.

Overall, we measure the robustness of the model based on a few aspects: different components in state-space models, number of AO added, values of target size, magnitude of AO, and locations of the AO in the series. We utilize the principles of potency and gauge to evaluate the effectiveness of the procedure for identifying outliers. Potency refers to the ratio of pertinent indicators that are retained in the ultimate model, whereas gauge pertains to the ratio of irrelevant indicators that persist in the ultimate model. Both potency and gauge are computed based on the retention rate, $\tilde{r}$ formulated as

$$\tilde{r} = \frac{1}{M}\sum_{i=1}^{M} 1\left[\widetilde{\beta_{ij}} \neq 0\right],\ \ j = 1, \ldots, T \qquad (9)$$

$$potency = \frac{1}{n}\sum_j \tilde{r}_j\ ,\ \ j \epsilon R_n \qquad (10)$$

$$gauge = \frac{1}{T-n}\sum_j \tilde{r}_j\ ,\ \ j \epsilon R_{T-n} \qquad (11)$$

where M denotes the number of replications and *n* denotes the number of true outliers in the time series of length T. $R_n$ and $R_{T-n}$ sets of time indices for relevant and irrelevant indicators retained in the model are denoted by and, respectively. The estimated coefficient in the impulse indicator is represented by $\widetilde{\beta_j}$, and if $I_t(k)$ is selected, then the variable $1[\widetilde{\beta_{ik}} \neq 0]$ will be assigned a value of one, indicating that the argument is true, and zero otherwise. To determine the value of target size $\alpha = \min[0.05, {}^{1}/_{T}]$, we adhere to the rule of thumb recommended by Pretis, Reade and Sucarrat (2018). This approach ensures that the gauge value remains low, below 5% of the sample size T, or that only one irrelevant indicator variable is kept in the final model. Loose value of $\alpha$ leads to the case of over-fittingmodel,whilestringentvalueof$\alpha$leadstothecaseof under-fitting model.

## MONTE CARLO SIMULATIONS RESULTS UNDER THE NULL HYPOTHESIS

This section aims to examine the performance of IIS and SIS in state-space models under the null hypothesis of no outliers. Overall, the Monte Carlo simulation experiment in this study consisted of two key parts. Firstly, this study evaluated the performance of IIS and SIS without the presence of outliers and structural breaks in the state space model framework. For the development of null hypotheses, this study proposed $H_0\!:\beta_t = 0, \forall t$ for IIS and $H_0\!:\delta_t = 0, \forall t$ for SIS. The selection of significant indicators retained in the model was based on $\alpha T$ at the selected level of significance, $\alpha$. Secondly, the Monte Carlo simulations are conducted with the presence of predetermined additive outliers in the state space model framework.

The series were initially generated for T = 150 observations. The first 30 observations were then removed to avoid the dependency on initial values, resulting in the following number of observations: T = 120 observations. Based on the obtained results, the retention frequencies of irrelevant indicators that were falsely retained in the model for the case of sequential selection algorithm were consistently close or below the considered level of significance. Such observation was apparent in all groups; the gauge values remained closer to and below the selected level of significance, α, particularly when α recorded 0.1% and 1/T. Based on the results from these Monte Carlo simulations, this study proved that the selected level of significance, α, controls the gauge values for each series generated in state-space models. It is important to select the appropriate α to reduce false-detection rate. Referring to Pretis, Reade and Sucarrat (2018), this study set $\alpha = \min\left\{\frac{1}{T}, 5\%\right\}$, targeting only one irrelevant indicator that was falsely retained in the model by chance. The current study's results also showed that the sequential selection algorithm produced consistently closer and lower gauge values to the selected α, as compared to the non-sequential selection algorithm. In other words, the performance of sequential selection algorithm surpasses the performance of non-sequential selection algorithm. For comparison, the case of sequential selection algorithm showed

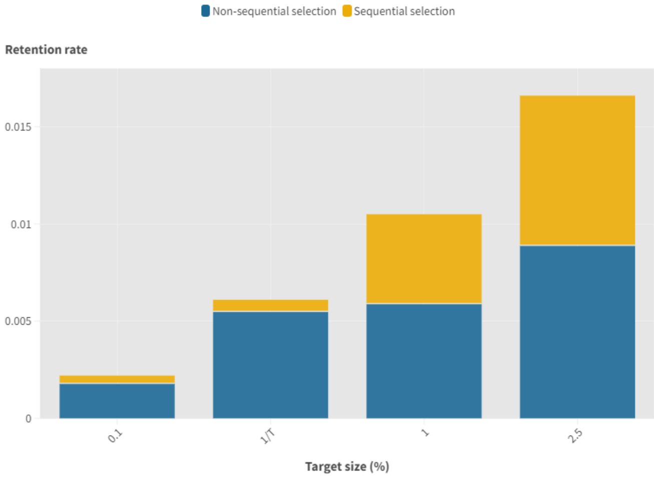


FIGURE 3. The retention rate of IIS+SIS for non-sequential and sequential selection algorithms

better performance, with consistent lower gauge values than $\alpha$. The case of non-sequential selection algorithm generated comparable performance, with certain gauge values exceeding the selected $\alpha$ in Figure 3. The use of non-sequential selection algorithm recorded gauge values with no differences of more than two-thousandths of percentage points. On the other hand, the use of sequential selection algorithm significantly reduced high retention frequencies, closer to $\alpha$. In other words, the use of sequential selection algorithm was deemed more fitting at a less stringent level of significance level (i.e., $\alpha \geq 5$).

## MONTE CARLO SIMULATIONS RESULTS BASED ON POTENCY AND GAUGE

The Monte Carlo results for $T = 120$ observations are summarised in Tables 1 and 2. Overall, IIS+SIS performed well in detecting almost 100% of $z$ values that are greater than 7 despite the inclusion of seasonal component. As we increased the target size value, the potency reached almost 100%. We split the sample to $m = 6$ blocks to estimate the model. Based on the Monte Carlo results shown in Tables 1 and 2, it is obvious that the performance of IIS+SIS relies heavily on the magnitude of outliers even though different target sizes were used. The potency when $z = 3$ is relatively low with a satisfactory gauge value. However, when the size of $z$ was increased to 7, the probability of the first detection is almost 100%. In non-sequential selection, number of blocks is a critical aspect that affected the performance of IIS+SIS in outlier and breaks detection. Thus, we decided to generate the results using six blocks for all generated series. A minimum number of blocks would minimise the risk of missing any essential structural changes. The potency achieved 100% when the size of AO is at least $z = 7$. This means that size of AO plays a vital role in outliers detection using both impulse and step indicators. When examined closely, we found that the average gauge values in sequential selection are much lower than in non-sequential selection. In fact, the gauge values for non-sequential selection exceed the sequential selection but still clustered around the significance level, $\alpha$ in most settings. Thus, sequential selection approach plays a crucial role in eliminating the indicator that is spuriously retained in the model. Through both cases of sequential and non-sequential selection algorithms, both types of indicators in this case clearly demonstrated good performance of detecting nearly 100% when $\lambda \geq 7\sigma$. As the level of significance was set more stringent, the use of sequential selection algorithm produced substantially lower gauge values. In other words, the selected level of significance influences gauge of both sequential and non-sequential selection algorithms despite the varying magnitudes of AO.

Based on Table 2, the results on the detection of double AOs showed similar patterns of potency, in which the performance of sequential selection algorithm surpassed the performance of non-sequential selection algorithm. When $\lambda = 3\sigma$, the potency values for the case of sequential selection algorithm were significantly different from the potency values for the case of non-sequential selection algorithm, but the differences did not exceed forty percentage points. Meanwhile, when $\lambda = 5\sigma$, the performance of IIS+SIS was found excellent, with average potency values of above 85% for both selection algorithms. Based on these patterns, the magnitude of AOs has influence on the performance of IIS in terms of potency. On the other hand, when it comes to gauge values, for the case of sequential selection algorithm demonstrated excellent performance, as shown in Table 2. The observed gauge values did not exceed the selected $\alpha$, which may be attributed to the use of multiple paths searching in reducing variances for estimators (Doornik, Hendry & Pretis 2013). For the case of non-sequential selection algorithm, IIS+SIS also showed excellent performance. The recorded gauge values were close around the selected α. The use of a more stringent level of significance is more likely linked to lower potency for coefficients and higher critical value, $c_a$. Meanwhile, when it comes to similar settings of λ = 3σ and 0.1% level of significance, the observed gauge in this study exceeded α by two-thousandths percentage points. This may be attributed to model misspecification, specifically the case of under-fitting. Therefore, these observations were deemed plausible and expected when the level of significance was set more stringent.

Tables 3 and 4 show the Monte Carlo results for single structural break and double structural breaks at different locations using six blocks estimation. The target size was chosen as $\alpha = 1/T$ and magnitude of breaks was 7σ. We found a more satisfactory potency values when the structural change is in the middle of the sample compared to nearby the end of the sample.

The combination of IIS and SIS recorded low gauge values that were close to the nominal level of significance, particularly when sequential selection algorithm was used. Based on these results, this study demonstrated the effectiveness of IIS and SIS in capturing irrelevant indicators in the model with respect to the null hypothesis. This study demonstrated the influence of the selected level of significance on the false-detection rate. The settings proposed by Pretis et al. (2016) were considered in the current study on the selection of appropriate level of significance, α = min {0.05, 1/T}. A higher level of significance (e.g., α ≤ 5%) potentially generates higher gauge values. Fundamentally, the selected level of significance ensures the formation of an over- or under-fitting model. Based on the results, it appears that an over-fitting model is more likely to occur at a less stringent level of significance, whereas an under-fitting model is more likely to occur at a more stringent level of significance. Considering numerous possible settings for the nominal level of significance, the current study recommended using the sequential selection algorithm when the selected level of significance is less stringent (e.g., 5%). Applying a more stringent level of significance also potentially results in lower potency for coefficients. When it comes to potency, the use of sequential selection algorithm produced better performance of IIS and SIS, as compared to the use of non-sequential selection algorithm when λ = 3σ. When λ = 5σ, the use of sequential selection algorithm significantly increased the average potency

TABLE 1. Performance evaluation of IIS+SIS detecting single AO at various significance level in state-space models

| Model | | Non-sequential | | | | | Sequential | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| LLM | α (%) | 3σ | 5σ | 7σ | 9σ | 12σ | 3σ | 5σ | 7σ | 9σ | 12σ |
| Potency (%) | 0.1 | 16.1 | 66.2 | 83.7 | 96.3 | 100 | 39.7 | 75.6 | 97.6 | 99.8 | 100 |
| | 1 | 47.1 | 89.5 | 99.1 | 100 | 100 | 50.1 | 92.5 | 99.7 | 99.9 | 100 |
| | 2.5 | 57.6 | 93.7 | 99.3 | 100 | 100 | 64.2 | 96.7 | 100 | 100 | 100 |
| Gauge (%) | 0.1 | 0.03 | 0.05 | 0.01 | 0.03 | 0.02 | 0.03 | 0.03 | 0.03 | 0.01 | 0.02 |
| | 1 | 0.93 | 0.87 | 0.86 | 0.97 | 0.82 | 0.58 | 0.47 | 0.52 | 0.41 | 0.36 |
| | 2.5 | 2.27 | 2.19 | 2.24 | 2.16 | 2.22 | 1.08 | 0.88 | 0.78 | 0.61 | 0.91 |
| LLTM | α (%) | 3σ | 5σ | 7σ | 9σ | 12σ | 3σ | 5σ | 7σ | 9σ | 12σ |
| Potency (%) | 0.1 | 23.7 | 70.7 | 90.1 | 99.7 | 99.9 | 47.9 | 81.8 | 98.9 | 99.9 | 100 |
| | 1 | 44.7 | 86.4 | 98.1 | 99.8 | 100 | 55.8 | 94.3 | 99.7 | 100 | 100 |
| | 2.5 | 59.3 | 92.1 | 99.0 | 99.9 | 100 | 69.7 | 96.7 | 99.9 | 100 | 100 |
| Gauge (%) | 0.1 | 0.21 | 0.19 | 0.15 | 0.10 | 0.11 | 0.08 | 0.05 | 0.02 | 0.00 | 0.00 |
| | 1 | 0.98 | 0.67 | 0.51 | 0.69 | 0.58 | 0.55 | 0.27 | 0.18 | 0.32 | 0.14 |
| | 2.5 | 2.03 | 1.69 | 1.72 | 1.67 | 1.74 | 1.56 | 0.28 | 1.16 | 1.15 | 1.29 |
| LLTSM | α (%) | 3σ | 5σ | 7σ | 9σ | 12σ | 3σ | 5σ | 7σ | 9σ | 12σ |
| Potency (%) | 0.1 | 17.9 | 69.5 | 88.1 | 96.2 | 99.7 | 67.1 | 88.5 | 98.4 | 99.6 | 100 |
| | 1 | 49.7 | 88.1 | 94.7 | 97.0 | 100 | 71.9 | 92.7 | 100 | 100 | 100 |
| | 2.5 | 38.8 | 83.7 | 98.6 | 98.3 | 100 | 73.7 | 96.4 | 100 | 100 | 100 |
| Gauge (%) | 0.1 | 0.23 | 0.20 | 0.14 | 0.08 | 0.02 | 0.06 | 0.09 | 0.05 | 0.03 | 0.01 |
| | 1 | 0.82 | 0.72 | 0.57 | 0.35 | 0.20 | 0.62 | 0.59 | 0.79 | 0.33 | 0.37 |
| | 2.5 | 2.51 | 2.67 | 2.55 | 2.49 | 2.36 | 1.22 | 1.76 | 1.59 | 1.44 | 1.27 |

to 90%. The performance of IIS in detecting relevant indicators was found to be perfect when $\lambda > 5\sigma$. In other words, higher magnitude of outliers and structural breaks enhances the effectiveness of IIS and SIS. This may be attributed to the iterative removal of the least significant indicators, which substantially lowers the variance of the estimator and number of irrelevant indicators retained and raises the retention frequencies of relevant indicators. Conclusively, the use of different variance parameters and number of observations in each DGP were found to have no influence on the effectiveness of the current study's IIS and SIS in capturing the true outlier. On the other hand, the magnitude of outliers and structural breaks clearly affected the potency of the study's IIS and SIS.

TABLE 2. Performance evaluation of IIS+SIS detecting double AO at various significance level in state-space models

| | | Non-sequential | | | | | Sequential | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| LLM | α (%) | 3σ | 5σ | 7σ | 9σ | 12σ | 3σ | 5σ | 7σ | 9σ | 12σ |
| Potency (%) | 0.1 | 19.2 | 74.6 | 94.7 | 99.6 | 100 | 55.3 | 84.7 | 98.7 | 99.8 | 100 |
| | 1 | 48.4 | 89.5 | 99.8 | 99.9 | 100 | 74.8 | 95.7 | 99.7 | 99.9 | 100 |
| | 2.5 | 68.5 | 93.1 | 98.6 | 100 | 100 | 79.3 | 97.5 | 100 | 100 | 100 |
| Gauge (%) | 0.1 | 0.05 | 0.07 | 0.08 | 0.07 | 0.05 | 0.04 | 0.02 | 0.03 | 0.02 | 0.00 |
| | 1 | 0.98 | 0.83 | 0.87 | 0.76 | 0.71 | 0.87 | 0.73 | 0.70 | 0.59 | 0.56 |
| | 2.5 | 1.98 | 1.84 | 1.10 | 0.52 | 0.14 | 1.02 | 1.12 | 1.28 | 1.16 | 1.22 |
| LLTM | α (%) | 3σ | 5σ | 7σ | 9σ | 12σ | 3σ | 5σ | 7σ | 9σ | 12σ |
| Potency (%) | 0.1 | 22.8 | 72.1 | 92.7 | 98.9 | 99.9 | 64.7 | 92.6 | 97.6 | 99.8 | 100 |
| | 1 | 49.7 | 89.2 | 98.8 | 99.9 | 100 | 77.3 | 92.4 | 99.7 | 100 | 100 |
| | 2.5 | 60.1 | 93.5 | 99.6 | 100 | 100 | 80.1 | 95.5 | 99.9 | 100 | 100 |
| Gauge (%) | 0.1 | 0.08 | 0.07 | 0.03 | 0.03 | 0.03 | 0.02 | 0.07 | 0.06 | 0.02 | 0.01 |
| | 1 | 0.84 | 0.69 | 0.51 | 0.49 | 0.22 | 0.78 | 0.61 | 0.33 | 0.26 | 0.16 |
| | 2.5 | 2.33 | 2.29 | 2.18 | 2.25 | 2.31 | 1.14 | 1.29 | 1.21 | 1.08 | 1.02 |
| LLTSM | α (%) | 3σ | 5σ | 7σ | 9σ | 12σ | 3σ | 5σ | 7σ | 9σ | 12σ |
| Potency (%) | 0.1 | 18.5 | 41.6 | 73.8 | 90.1 | 98.5 | 67.9 | 93.2 | 98.0 | 100 | 100 |
| | 1 | 25.1 | 68.5 | 89.9 | 96.4 | 99.4 | 48.6 | 92.7 | 99.3 | 100 | 100 |
| | 2.5 | 44.6 | 75.9 | 92.0 | 97.8 | 99.9 | 85.4 | 98.3 | 100 | 100 | 100 |
| Gauge (%) | 0.1 | 0.07 | 0.06 | 0.07 | 0.05 | 0.01 | 0.07 | 0.09 | 0.04 | 0.00 | 0.00 |
| | 1 | 1.12 | 1.02 | 1.09 | 0.78 | 0.59 | 0.21 | 0.11 | 0.18 | 0.37 | 0.15 |
| | 2.5 | 3.28 | 3.74 | 3.13 | 3.28 | 3.05 | 0.18 | 0.28 | 0.33 | 0.21 | 0.83 |

TABLE 3. Potency and gauge values for single structural breaks at various locations

| Location of AO | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 |
|---|---|---|---|---|---|
| Potency (%) | 97.9 | 98.9 | 99.5 | 98.9 | 97.9 |
| Gauge (%) | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |

TABLE 4. Potency and gauge values for double structural breaks at various locations

| Location of double AO | [0.1,0.2] | [0.3,0.4] | [0.5,0.6] | [0.7,0.8] | [0.9,1] |
|---|---|---|---|---|---|
| Potency (%) | 99.5 | 99.05 | 99.65 | 98.25 | 97.35 |
| Gauge (%) | 0.3525 | 0.1780 | 0.0805 | 0.1237 | 0.3576 |

## EMPIRICAL APPLICATIONS

Detection of outliers has essential effects on economic time series data for parameter estimation and forecasting purposes. We apply the indicator saturation approach to the monthly FTSE 100 closing stock price index obtained from Datastream. The reference model framework for this application is the local linear trend model with seasonal component. The data covers the period from October 2013 until October 2023, consisting of 120 observations, T.

This section presents the results of diagnostics tests following the application the structural time series model using actual data. Firstly, the FTSE 100 stock price were analysed based on Akaike information criterion (AIC) and Bayesian information criterion (BIC) to examine whether the data fit to the structural time series model. For the analysis, return values, $r_t$, of the FTSE 100 closed price data were determined from the log difference of monthly stock prices. Accordingly, residuals in the structural time series model are generally assumed to be independent and normally distributed, with the attribute of homoscedasticity. Therefore, the current study performed the following diagnostics tests to examine whether the residuals meet these respective properties: (1) Durbin-Watson test; (2) homoscedasticity test; (3) normality test. Overall, the diagnostic tests results appear satisfactory for every model. It can be seen that most of the values of autocorrelations at lag 1 converge to zero indicating weak positive correlation among residuals. Moreover, the Durbin-Watsons statistics values are clustered around 2 indicating the same correlation between residuals. The H-statistics indicate the variances of two consecutive and equal parts of the residuals are equal. For instance, in Table 5, the test shows that the variance of the 52 elements of the residuals is unequal to the variance of the last 52 elements of the residuals. Summarising, the assumptions of independence, homoscedasticity and normality are all satisfied for FTSE 100. When comparing the info criterion values, this study holds the rule of thumb: the smaller values denote better fitting models than larger ones. Overall, the AIC and BIC values are approximately the same for the local level model.

The selection of significant level is governed by $^1/_T$ which manifests that generally less than one indicator being remained spuriously under the null of no outliers. We split the blocks into six with multi-path indicator saturation. The objective of this application is to assess how indicator saturation, specifically IIS+SIS depicts recessionary events triggered by financial crises around the world, especially during the COVID-19 pandemic in 2020. As expected, the results show that structural changes are detected during the year 2020. Interestingly, we found that the structural changes detected in 2020 are negatively associated with the global economic recession that occurred due to COVID-19 pandemic. This result is consistent with previous occurrences of financial crises, which IIS+SIS interprets as recessions.

This study postulates that the structural breaks will be detected in FTSE 100 during the year 2020 as shown in Figure 4. Such a result should not come as a surprise given the major regime changes happened globally over the period of 2020–2021. Hence, parameter estimation in GETS modelling can be done by taking into account the outliers detected by IIS and SIS in the model estimation. This particular study distinguishes itself from the research conducted by Bakar (2019) due to the presence of methodological deficiencies. These deficiencies were mainly attributed to the Box-Whisker plot's inability to effectively handle non-stationary data and adequately represent the structural change identified in the FTSE 100. Furthermore, in contrast to the IIS+SIS in GETS modelling, the Box-Whisker plot approach lacks the capability to conduct significance testing. Hence, the final selected model is reported using the *Autometrics* algorithm. The procedure of model selection began with the general unrestricted model, then selecting the significant regressors which will be retained in the model. The model estimation was over the period October 2013 until October 2023 where the impulse indicators as $\{1\{j = t\}\}$ where $\{1\{j = t\}\}$ corresponds to one when $j = t$ and equal to zero otherwise for $j = 1, \ldots, T$. Meanwhile the steps indicators is in the form $\{1\{j \geq t\}\}$. This research estimates the simple model of closed price for FTSE 100 as in Equation (6) including the IIS+SIS at a target significance level $\alpha = {}^1/_T$. This implies that an expected of false positive the number of breaks detected of $1 / 120 \approx 0.0083$ outlier. The resulting model captured two breaks in the log of monthly closed price for FTSE 100 in the year 2020.

$$\begin{gathered}\log(y_t) = \tilde{\mu}_{t|T} + 0.16\delta_{t \geq 2020.M1} - 0.29\delta_{t \geq 2020.M3} + \\ 0.09\delta_{t \geq 2020.M7} - 0.11\delta_{t \geq 2020.M9} + \tilde{\varepsilon}_{t|T} \\ R^2_{adj} = 0.4019, \quad Normality = 1.289, \\ H(34) = 1.7194, \quad DW = 1.9856\end{gathered} \tag{12}$$

where $y_t$ is the log of monthly closed price for FTSE 100, $\tilde{\mu}_{t|T}$ and $\tilde{\varepsilon}_{t|T}$ are the smoothed estimates of the trend and irregular components. Referring to Equation (12), the coefficients for the structural breaks at $t = 2020.M1$ and $t = 2020.M3$ are estimated at $\hat{\beta}_{t=2020.M3} = 0.16$ and $\hat{\beta}_{t=2020.M9} = -0.19$, respectively. The magnitude of the structural breaks was quantified based on the estimated prediction error variance of the model, $\hat{\sigma}_\varepsilon = 0.0019$. The magnitude of structural break is expressed in term of estimated error variance, $\delta = {}^{\hat{\beta}}/_{\hat{\sigma}_\varepsilon}$. Hence, the two structural breaks detected has magnitude of $84.21\hat{\sigma}_\varepsilon$ and $152.63\hat{\sigma}_\varepsilon$. Likewise, the coefficients for the structural breaks at $t = 2020.M7$ and $t = 2020.M9$ are estimated at $\hat{\beta}_{t=2020.M7} = 0.09$ and $\hat{\beta}_{t=2020.M9} = -0.11$, respectively. Thus, the breaks have magnitude of $47.36\hat{\sigma}_\varepsilon$ and $57.89\hat{\sigma}_\varepsilon$. Overall, this finding highlights the novel aspect of this research to identify the magnitude, location, and sign of the breaks immediately at the start of the sample

TABLE 5. Diagnostic statistics tests results for FTSE 100 closed price

| | Statistics | LLM |
|---|---|---|
| Independence | DW | 1.766 |
| | r(1) | 0.1032 |
| Homoscedasticity | H(h) | H(52) 2.084 |
| Normality | N | 70.929 |
| Information Criterion | AIC | 7.4701 |
| | BIC | 7.5088 |

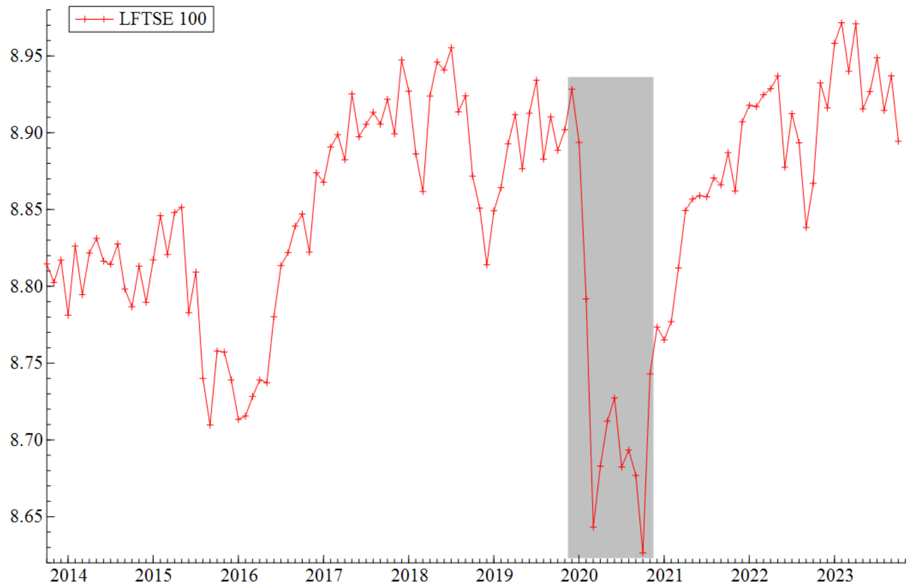


FIGURE 4. Log of monthly closed price of FTSE 100 from October 2013 to October 2023

observation until the end in state-space model framework. This finding may lead to better understanding of how the stock markets in the UK reacts to government policy due to the COVID-19 pandemic. The results also indicate that it is possible to quantify the impact of a structural change at a specific date on the stock markets.

## CONCLUSIONS

Our study aimed to examine the ability of impulse and steps indicator saturation in detecting structural breaks in state-space models. The performance of IIS and SIS was measured using potency and gauge via extensive Monte Carlo experiments. Hence, we conclude that IIS+SIS are very useful in detecting structural breaks. This research has made a substantial contribution to providing a novel approach for detecting structural breaks and outliers simultaneously.

We discovered a few aspects that can affect the performance of IIS+SIS. First, the size and magnitude of AO. IIS becomes more effective as the size of AO increases starting from $z = 5$. Secondly, the target size chosen also affects the potency value as it determines the number of irrelevant indicators to be retained in the model. Next, the number of blocks is also an important factor in IIS+SIS performance. Fourth, IIS+SIS performs better in detecting single AO compared to double AO in terms of potency above 80%. Finally, the location of AO plays a vital role in the performance of IIS+SIS. We found that the potency achieved its maximum of 100% when the location of AO is in the middle of the sample. In the last part of the work, we applied IIS+SIS to the monthly stock returns of FTSE 100 with the aim to investigate the application of IIS+SIS to depict the global recession movement that affected the FTSE 100. Overall, IIS+SIS is proven effective in detecting structural breaks in the state-space model. Even though IIS is initially designed to detect outliers, it is also capable to detect single location shift using split-half approach when a single location shift exists in a series.

Further research might delve into quantitative comparisons with other models, refining the algorithm's parameters to accommodate varying data characteristics, contributing to its versatility in anomaly detection scenarios. Finally, trend indicator saturation (TIS) may be utilized in state spaces models to capture any structural change that happened.

ACKNOWLEDGEMENTS

We would like to thank the reviewer for the constructive feedback. We also gratefully acknowledged the financial support from Universiti Putra Malaysia for Inisiatif Putra Muda Grant GP-IPM/2023/9773500.

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