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EFFECTS OF SUCTION AND INJECTION ON THE STAGNATION POINT FLOW OVER A STRETCHING SHEET IN A MICROPOLAR FLUID

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ABSTRACT

An analysis is made for the steady two-dimensional stagnation-point flow of a micropolar fluid over a stretching permeable sheet in its own plane. The stretching velocity and the ambient fluid velocity are assumed to vary linearly with the distance from the stagnation-point. The transformed boundary layer equations are solved numerically for some values of the involved parameters using a finite-difference method. The features of the flow characteristics are analyzed and discussed. Comparison with the existing results for some particular cases of the present study has been done and found to be in excellent agreement. It is also observed that suction increases the skin friction coefficient whereas injection decreases it.

*Keywords:*boundary layer; micropolar fluid; stagnation-point flow; stretching surface; suction/injection

1. Introduction

Suction or injection (blowing) of a fluid through the bounding surface, as, for example, in mass transfer cooling, can significantly change the flow field and, as a consequence, affects the heat transfer rate from the surface. In general, suction tends to increase the skin friction and heat transfer coefficients, whereas injection acts in the opposite manner (Al-Sanea 2004). Injection or withdrawal of fluid through a porous bounding heated or cooled wall is of general interest in practical problems involving boundary layer control applications such as film cooling, polymer fiber coating, coating of wires, etc. The process of suction and blowing has also its importance in many engineering activities such as in the design of thrust bearing and radial diffusers, and thermal oil recovery. Suction is applied to chemical processes to remove reactants. Blowing is used to add reactants, cool the surface, prevent corrosion or scaling and reduce the drag (Labropulu 1996).

Crane (1970) was the first to study the steady two-dimensional boundary layer flow due to a stretching surface in a viscous and incompressible quiescent fluid, and presented an exact analytical solution. Subsequently, many authors have considered various aspects of this problem and obtained similarity solutions. The papers by Magyari and Keller (1999, 2000, 2005), and Liao and Pop (2004) contain a good amount of references on this problem. Chiam (1994) and Mahapatra and Gupta (2002) have investigated the steady two-dimensional stagnation point flow of an incompressible viscous fluid over a flat deformable sheet when the sheet is stretched in its own plane with a velocity proportional to the distance from the stagnation point. It is shown that a boundary layer is formed near the stretching surface and that the structure of this boundary layer depends on the ratio of the velocity of the stretching surface to that of the frictionless potential flow in the neighborhood of the stagnation point. This problem was then extended by Nazar et al. (2004)

to the boundary layer flow past a stretching sheet in an incompressible micropolar fluid. Further, Hayat et al. (2008) extended the work of Nazar et al. (2004) by including the effect of magnetic parameter M on the flow field, and solved the problem analytically using the Homotopy Analysis Method. Unfortunately, Eq. (5) in that paper was not correctly derived and thus the results presented are not accurate when the magnetic parameter $M \neq 0$. The objective of the present paper is not to criticize the paper by Hayat et al. (2008), but to extend the work of Nazar et al. (2004) by including the effects of suction and injection on the wall surface which influence the flow field significantly.

2. Problem Formulation

Consider the flow of an incompressible micropolar fluid in the region $y > 0$ driven by a stretching surface located at $y = 0$ with a fixed stagnation-point at $x = 0$. The stretching velocity $u_w(x)$ and the ambient fluid velocity $u_e(x)$ are assumed to vary proportional to the distance x from the stagnation-point, i.e. $u_w(x) = ax$ and $u_e(x) = bx$, where a and b are constants with $a > 0$ and $b \geq 0$. The simplified two-dimensional equations governing the flow in the boundary layer of a steady, laminar and incompressible micropolar fluid are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + (\nu + \frac{\kappa}{\rho})\frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho}\frac{\partial N}{\partial y}, \tag{2}$$

$$\rho j(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}) = \gamma\frac{\partial^2 N}{\partial y^2} - \kappa(2N + \frac{\partial u}{\partial y}), \tag{3}$$

subject to the boundary conditions

$$u = u_w(x),\ v = v_w,\ N = -m\frac{\partial u}{\partial y} \quad \text{at} \quad y = 0,$$

$$u \to u_e(x),\ N \to 0 \quad \text{as} \quad y \to \infty, \tag{4}$$

where u and v are the velocity components along the x and y axes, respectively, N is the microrotation or angular velocity whose direction of rotation is normal to the $x - y$ plane, ν is kinematic viscosity, ρ is fluid density, j is microinertia per unit mass, γ is spin gradient viscosity, and κ is vortex viscosity. Further, $v_w < 0$ and $v_w > 0$ are for mass suction and mass injection, respectively. We notice that m is a constant such that $0 \leq m \leq 1$. The case $m = 0$, which indicates $N = 0$ at the surface represents concentrated particle flows in which the microelements close to the wall surface are unable to rotate (Jena & Mathur 1981). This case is also known as strong concentration of microelements (Guram & Smith 1980). The case $m = 1/2$ indicates the vanishing of anti-symmetric part of the stress tensor and denotes weak concentration (Ahmadi 1976) of microelements. The case $m = 1$ as suggested by Peddieson (1972) is used for the modeling of turbulent boundary layer flows. In this paper we consider only the case $m = 1/2$. Following Ahmadi (1976), we assumed that γ is given by

$$\gamma = \left(\mu + \frac{\kappa}{2}\right) j \tag{5}$$

where $\mu = \rho\nu$ is the dynamic viscosity, and we take $j = \nu/a$ as a reference length. Relation (5) is invoked to allow Eqs. (1)-(4) to predict the correct behavior in the limiting case when microstructure effects become negligible, and the microrotation N reduces to the angular velocity.

The governing equations (1)-(3) subject to the boundary conditions (4) can be expressed in a simpler form by introducing the following transformation:

$$\eta = \left(\frac{a}{\nu}\right)^{1/2} y, \quad u = axf'(\eta), \quad v = -(a\nu)^{1/2} f(\eta), \quad N = ax\left(\frac{a}{\nu}\right)^{1/2} g(\eta), \tag{6}$$

to obtain the following ordinary differential equations:

$$(1 + K)f''' + ff'' - f'^2 + \varepsilon^2 + Kg' = 0, \tag{7}$$

$$(1 + K/2)g'' + fg' - f'g - K(2g + f'') = 0, \tag{8}$$

where $K = \kappa/\mu$ (≥ 0) is the material parameter, $\varepsilon = b/a$ is the velocity ratio parameter and primes denote differentiation with respect to η. The boundary conditions (4) now become

$$\begin{gathered} f(0) = f_0,\ f'(0) = 1,\ h(0) = -mf''(0), \\ f'(\eta) \to \varepsilon,\ g(\eta) \to 0, \quad \text{as} \quad \eta \to \infty, \end{gathered} \tag{9}$$

where $f_0 = -v_w/(\nu a)^{1/2}$ is a constant which determines the transpiration rate at the surface, with $f_0 > 0$ for suction, $f_0 < 0$ for blowing or injection, and $f_0 = 0$ corresponds to an impermeable plate.

The physical quantity of interest is the skin friction coefficient C_f, which is defined as

$$C_f = \frac{\tau_w}{\rho u_w^2}, \tag{10}$$

where τ_w is the wall shear stress and is given by

$$\tau_w = \left[(\mu + \kappa)\frac{\partial u}{\partial y} + \kappa N\right]_{y=0}. \tag{11}$$

Using variables (6), we obtain

$$C_f Re_x^{1/2} = [1 + (1 - m)K]\, f''(0), \tag{12}$$

where $Re_x = u_w x/\nu$ is the local Reynolds number.

3. Results and discussion

Eqs. (7) and (8) subject to the boundary conditions (9) have been solved numerically using the Keller-box method, which is described in the book by Cebeci and Bradshaw (1988). This method has also been very successfully used by the present authors for other problems in micropolar fluids (Ishak et al. 2008a,b). In order to validate the accuracy of the present results, we have compared our results with those of Mahapatra and Gupta (2002), Nazar et al. (2004) and Hayat et al. (2008) for the case of impermeable surface as presented in Table 1, which showed an excellent agreement. Therefore, it may be concluded that the developed code can be used with great confidence to study the problem discussed in this paper.

Table 1: Values of the skin friction coefficient $C_f Re_x^{1/2}$ for various values of ε when $K = 0$ and $f_0 = 0$

ε	Mahapatra and Gupta (2002)	Nazar et al. (2004)	Hayat et al. (2008)	Present results
0.1	-0.9694	-0.9694	-0.96938	-0.969387
0.2	-0.9181	-0.9181	-0.91810	-0.918108
0.5	-0.6673	-0.6673	-0.66732	-0.667265
1	–	0.0000	0.00000	0.000000
2	2.0175	2.0175	2.01750	2.017531
3	4.7293	4.7296	4.72928	4.729283

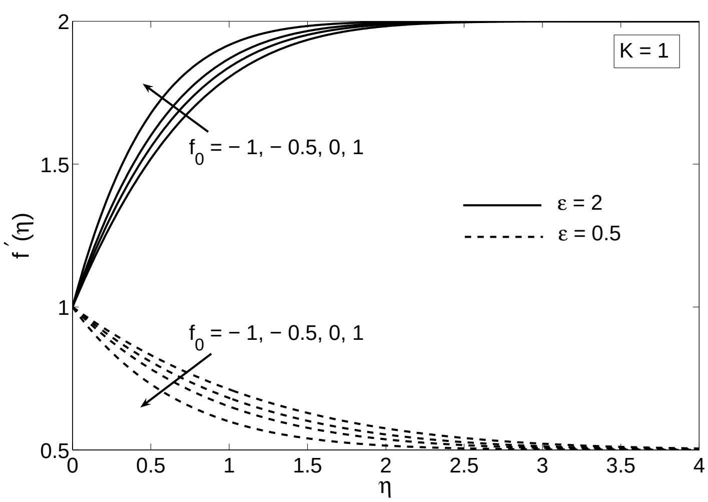

Figure 1: Velocity profiles $f'(\eta)$ for different values of ε and f_0 when $K = 1$

In the following discussion, we consider only the value of $K = 1$, as the results are similar for other values of K, as reported by Nazar et al. (2004). Fig. 1 illustrates the distribution of the velocity profiles $f'(\eta)$ for different values of f_0 and ε. It is observed that for $\varepsilon = 2$, the flow has a boundary layer structure and the thickness of the boundary layer decreases with an increase in f_0. In contrast, for $\varepsilon = 0.5$, the flow has an inverted boundary layer structure which results from the fact that when $\varepsilon\ (= b/a) < 1$, the stretching velocity ax of the surface exceeds the velocity bx of the free stream. For both cases $\varepsilon = 0.5$ and $\varepsilon = 2$, the boundary layer thickness decreases with an increase in f_0, which implies in increasing manner of the magnitude of the velocity gradient at the surface, and thus increases the magnitude of the wall shear stress with f_0. Hence, suction increases the skin friction, whereas injection decreases it.

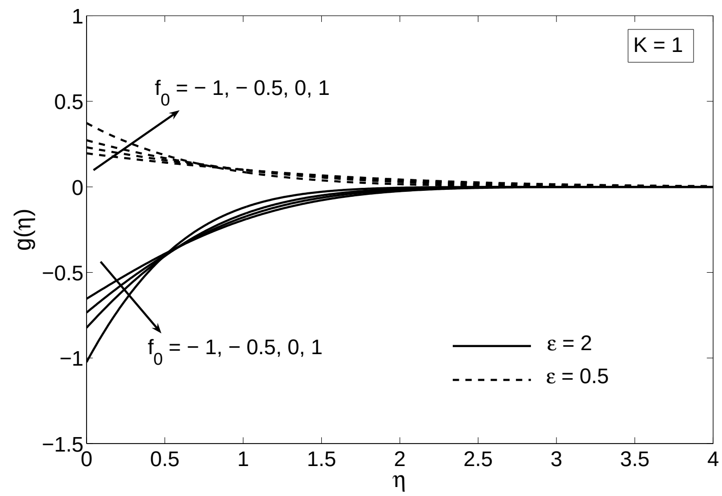

Figure 2: Angular velocity profiles $g(\eta)$ for different values of ε and f_0 when $K = 1$

Fig. 2 presents the microrotation or angular velocity $g(\eta)$ for various values of f_0 and ε. The magnitude of the microrotation continuously decreases with η for both $\varepsilon = 0.5$ and $\varepsilon = 2$, and becomes zero far away from the plate, which satisfies the boundary conditions (9). As expected, the microrotation effects are more dominant near the wall. Also, the magnitude of the microrotation increases as f_0 increases in the vicinity of the surface but the reverse happens as one moves away from it.

The variations of $C_f Re_x^{1/2}$ as a function of ε for various values of the suction/injection parameter f_0 are shown in Fig. 3. It is evident from this figure that, all curves intersect at a point where $\varepsilon = 1$, that is when the fluid velocity equals the velocity of the stretching sheet. In this case, the skin friction $C_f Re_x^{1/2} = 0$ since there is no wall shear stress $(\tau_w = 0)$ when the sheet and the fluid move with the same velocity. Further, it can be observed that increasing f_0 results in increasing manner of the magnitude of the skin friction coefficient, which is consistent with the results presented in Fig. 1. The value of $C_f Re_x^{1/2}$ is negative when $\varepsilon < 1$ but it is positive when $\varepsilon > 1$. This is not surprising since the fluid velocity exceeds the surface velocity when $\varepsilon > 1$, and the opposite is true when $\varepsilon < 1$. Physically, positive sign of the skin friction coefficient means the fluid exerts a drag force on the sheet, while negative sign means the opposite.

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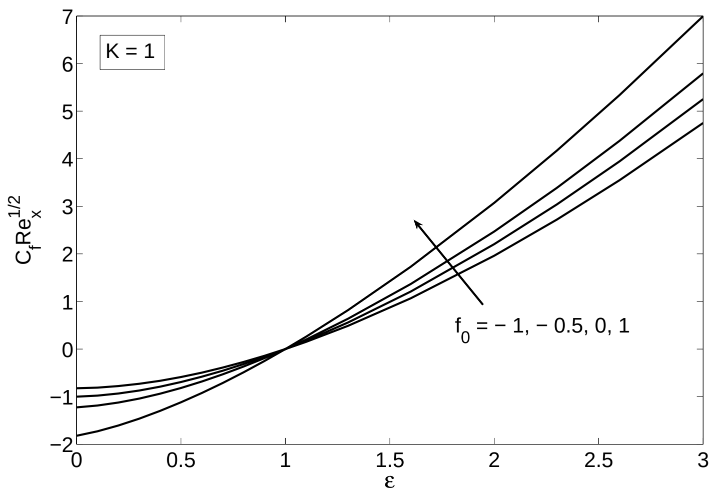

Figure 3: Variation of the skin friction coefficient $C_f Re_x^{1/2}$ with ε for various values of f_0 when $K = 1$

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